USING THE SOLAR ECLIPSE TO GAIN INSIGHT INTO THE SOLAR NEUTRINO PROBLEM

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Abstract

Currently the flux of solar [electron] neutrinos detected at real time, low energy neutrino detectors here on earth is significantly lower than predicted. The proposed, and recently confirmed, solution to this Solar Neutrino Problem is the process of neutrino oscillations. Such oscillations (also referred to as flavour mixing) are said to be enhanced by the presence of matter. This is known as the MSW Effect. In the event that the moon obstructs the path of solar neutrinos on their way to earth, as during solar eclipses, then we should be able to use our knowledge of the MSW Effect to gain further insight into the Solar Neutrino Problem. In this thesis we will examine the nature of nuetrinos. We will also examine the process of flavour mixing and derive formulae for the probability that an electron neutrino will change flavours in the moon. In addition, as we will need to know the percent coverage of each eclipse at each neutrino detector site of interest, projected eclipse data for the years 2003-2035 will also be provided.

I. Introduction

Neutrinos are electrically neutral particles of spin 1/2. They were proposed to account for difficulties in studies of beta radiation, a nuclear decay process in which the charge of the nucleus is increased by one and a "beta ray" (or electron) is emitted. However, the mass energy of the system was apparently not being conserved. The solution is the hypothesis of a neutral particle which is emitted in conjunction with the electron. Today we know these particles as neutrinos.

According to the Standard Model, neutrinos are massless particles. There are three "flavours" or distinct types of neutrinos: electron neutrinos, muon neutrinos, and tau neutrinos. The flavours derive their names from the particles they interact with, namely the electron, muon, and tau particle respectively. Electron neutrinos are the only flavour generated in, and hence emitted from, the core of the sun. These low-energy neutrinos propagate through space and their flux can be observed by means of solar neutrino detectors (e.g. Super-K in Kamioka, Japan). However, analysis of solar neutrino detection results poses a significant problem. The "Solar Neutrino Problem" (or SNP) is the discrepancy between the measured and predicted solar neutrino flux. Approximately 37% of the expected value[1] (depending on the detectable energy range each detector can observe) of electron neutrinos originating at the sun's core are being detected here on Earth.

One of the initial solutions proposed was that of flavour oscillations; neutrinos "oscillate" or change flavours as they travel from the sun's core toward the earth. Oscillations between flavours necessitate that neutrinos have mass. This disagrees with the Standard Model which predicts neutrinos to be massless. In June of 2001 the Sudbury Neutrino Observatory (SNO) in Sudbury, Canada released their results which confirmed flavour oscillations [2]. Consequently the Standard Model of the Universe for elementary particles needs some adjustments in order to account for all known properties of neutrinos.

Neutrino oscillations, and hence detection rates as well, vary depending upon the medium through which the neutrinos propagate. Specifically, oscillations are enhanced in the presence of matter. This is known as the MSW Effect due to the contributions of S.P. Mikheev, A.Yu. Smirnov, and L. Wolfenstein on this subject. First observed by Wolfenstein in 1978, it was noted that if neutrinos have the ability to change flavours, such a change can be magnified as the neutrinos propagate through matter. Both the moon (during solar eclipses) and the earth (at night and during "double eclipses") pose as convenient sources of matter with which to obstruct neutrinos along their path from the sun's core to the earth. In addition, the matter outside the sun's core that neutrinos must traverse through before entering space will also effect the oscillations and hence the final detection rate of electron neutrinos. It will be necessary to take these effects into account in order to make any accurate calculations.

The day/night effect refers to the role the mass of the earth plays in the solar neutrino flux observed for detector sites at night, when the sun is no longer visible. In such instances, solar neutrinos must travel through the earth to reach the detectors. The SNO Collaboration has studied this phenomenon in great detail. Using the day/night effect data, the notion of flavour oscillations has been proven to be the solution to the SNP with the probability that oscillations occur being 99.9999% [2].

Observation of the effect of solar eclipses (i.e. the moon effect) is a means of further examining the phenomenon of flavour mixing. The moon effect is only relevant when the moon obstructs the path of solar neutrinos en route from the center of the sun to the earth. Recall that electron neutrinos alone are generated in the core of the sun. All solar neutrinos which are detected, by definition, began their journey at the center of the sun. We can assume that these particles travel in a straight line radially outward from the sun as they propagate through space. Therefore it is necessary for observation of the moon effect that the moon block from view (at the detector site) the center of the sun. To ensure that this is the case, at least 39% of the sun's visible area (we treat the sun as a two-dimensional disc) must be covered by the moon [3].

At present there are six sites at which solar neutrino detectors exist or have been proposed [4]. As the capabilities and accuracy of detectors continue to improve, so too will our understanding of the fundamental nature of neutrinos. One of the goals of this project is to determine dates and times of solar eclipses for which it may be possible to observe the MSW Effect at each of these sites.

In Section II, the nature of neutrino oscillations will be examined thoroughly. Time propagation and the MSW Effect will be taken into account in order to derive the probability of an electron neutrino generated in the core of the sun surviving until detection. We will also examine the flavour mixing parameters in matter in terms of the vacuum mixing angle θ_o and the squared difference of the masses Δm_o^2, the free flavour mixing parameters in a vacuum. In Section III, data regarding experimental ranges of the free parameters will be discussed as will resonance conditions. Section IV will contain the necessary astronomy needed for use in the calculations, such as the time dependence of the magnitude of solar eclipses and the relationship between the coordinates of single eclipses and double eclipses. Note that "double eclipse" refers to the event in which, if the earth were transparent, the eclipse would be visible from the detector site; it is referred to as a double eclipse due to the fact that the neutrinos must traverse through both the moon and the earth in order to reach the detector. Section V will focus on the detectors themselves, their current or proposed locations, as well as specifically when a solar eclipse of magnitude greater than 39% will have an effect. These dates have been determined through the year 2035. Calculations with respect to expected drop in solar neutrino flux at each detector site for each eclipse of note will be computed and discussed in Section VI. Finally, Section VII will provide a summary of the content of this thesis.

II. Neutrino Oscillations

As stated in the Introduction, only a small percentage of the expected flux of electron neutrinos is observed. One proposed solution was that of flavour mixing, a.k.a. neutrino flavour oscillations. This phenomenon occurs when neutrinos, propagating through space on their way to earth, oscillate between neutrino types. Initially at the core of the sun electron neutrinos are the only flavour present. If oscillations occur, both electron and muon neutrinos will be present when the neutrinos reach the earth. Thus, only a percentage of the original electron neutrinos leaving the sun's core can be recognized. Flavour mixing has been substantiated as the solution to the SNP [2]. Now let us step through the calculations which provide the foundations for neutrino oscillations as well as the effect that a solar eclipse will have on them.

There are two bases for neutrinos: the mass basis and the flavour basis. The flavour basis is given in terms of neutrino flavour. Recall from Section I that neutrino flavour simply distinguishes neutrinos based on which particle (i.e. electron, muon, and tau particles) it will interact with. The mass basis is given in terms of the mass eigenstates which are measurable quantities. The flavour basis can be written in terms of the mass basis, and hence the mass basis has three elements as well. However, we will only be working with two-element bases here. The reason is that transitions into tau neutrinos are excluded based on their energy. The solar neutrinos are low energy (1-15 MeV), and hence oscillations under analysis are restricted to those between electron neutrinos and muon neutrinos. This restricts us to a two-element mass basis as well.

The mass basis is defined and can be determined from the Schrodinger equation below:

(1) $$i\left(\frac{\partial}{\partial t}\begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix}\right)=\begin{bmatrix} \sqrt{K^2+m_1^{\ 2}} & 0 \\ 0 & \sqrt{K^2+m_2^{\ 2}} \end{bmatrix}\begin{bmatrix} \nu_1(0) \\ \nu_2(0) \end{bmatrix}$$

where K is the energy and m_i is the mass which determine the mass eigenvalues above. As a result of the fact that neutrinos are relativistic particles, we can discard most terms of the Taylor expansion of the diagonal entries in the Hamiltonian above. Then we can approximate the terms in the form of

$\sqrt{K^2 + m_i^2}$ to be $\frac{m_i^2}{2K} + K$ and the solution to the Schrodinger equation in the mass basis is simply:

(2)
$$\begin{bmatrix} \nu_1(t) \\ \nu_2(t) \end{bmatrix} = \begin{bmatrix} e^{\left(-\left[\frac{i m_1^2 t}{2K} + i K t\right]\right)} & 0 \\ 0 & e^{\left(-\left[\frac{i m_2^2 t}{2K} + i K t\right]\right)} \end{bmatrix} \begin{bmatrix} \nu_1(0) \\ \nu_2(0) \end{bmatrix}$$

The relationship between the flavour basis and the mass basis is given by the rotation below:

(3)
$$\begin{bmatrix} \nu_e(\tau) \\ \nu_\mu(\tau) \end{bmatrix} = \begin{bmatrix} \cos(\theta_o) & -\sin(\theta_o) \\ \sin(\theta_o) & \cos(\theta_o) \end{bmatrix} \begin{bmatrix} \nu_1(\tau) \\ \nu_2(\tau) \end{bmatrix}$$

The subscripts of θ and Δm^2 in (3) correspond to propagation in a vacuum.

The probability that an electron neutrino which originates in the sun has oscillated into a muon neutrino at time τ, when it reaches the detector, is what we will now examine. The probability that we are interested in then follows below, where we have substituted L for t for relativistic particles:

$$P(\nu_e(0) \rightarrow \nu_\mu(\tau)) = |< \nu_\mu(\tau) \mid \nu_e(0) >|^2:$$

For Interactions In a Vacuum:

Using the results from (2) and plugging into the rotation matrix (3) we can solve for the following:

$$< \nu_\mu(t) \mid \nu_e(0) > = \sin(\theta_o)\cos(\theta_o) < \nu_1(t) \mid \nu_1(0) > - \sin(\theta_o)\cos(\theta_o) < \nu_2(t) \mid \nu_2(0) >$$

$$= \sin(\theta_o)\cos(\theta_o)\, e^{(-iKt)}\, e^{\left(\frac{i m_1^2 L}{2K}\right)} < \nu_1(0) \mid \nu_1(0) >$$

$$- \sin(\theta_o)\cos(\theta_o)\, e^{(-iKt)}\, e^{\left(\frac{i m_2^2 L}{2K}\right)} < \nu_2(0) \mid \nu_2(0) >$$

$$= \sin(\theta_o)\cos(\theta_o)\, e^{(-iKt)}\left[\left[\cos\left(\frac{m_1^2 L}{2K}\right) - \cos\left(\frac{m_2^2 L}{2K}\right)\right] - i\left[\sin\left(\frac{m_1^2 L}{2K}\right) - \sin\left(\frac{m_2^2 L}{2K}\right)\right]\right]$$

(4)
$$= \left[-\frac{1}{2}\right]\sin(2\theta_o)\, e^{(-iKt)}\left[\left[\cos\left(\frac{m_2^2 L}{2K}\right) - \cos\left(\frac{m_1^2 L}{2K}\right)\right] - i\left[\sin\left(\frac{m_2^2 L}{2K}\right) - \sin\left(\frac{m_1^2 L}{2K}\right)\right]\right]$$

Therefore, the probability is simply the above value multiplied by its complex conjugate:

$P(\nu_e(0) \rightarrow \nu_\mu(\tau))$:

$$= \left[\frac{1}{4}\right]\sin^2(2\theta_o)\left[\left[\cos\left(\frac{m_2^2 L}{2K}\right) - \cos\left(\frac{m_1^2 L}{2K}\right)\right]^2 + \left[\sin\left(\frac{m_2^2 L}{2K}\right) - \sin\left(\frac{m_1^2 L}{2K}\right)\right]^2\right]$$

$$= \left[\frac{1}{4}\right]\sin^2(2\theta_o)\left[2 - 2\left(\cos\left(\frac{m_2^2 L}{2K}\right)\cos\left(\frac{m_1^2 L}{2K}\right) + \sin\left(\frac{m_2^2 L}{2K}\right)\sin\left(\frac{m_1^2 L}{2K}\right)\right)\right]$$

(5)
$$= \sin^2(2\theta_o)\sin^2\left[\frac{m_2^2 L}{4K} - \frac{m_1^2 L}{4K}\right]$$

In addition, if we define the quantity $\Delta m_o^2 = m_2^2 - m_1^2$, then our result simplifies to:

(6)
$$P(\nu_e(0) \rightarrow \nu_\mu(\tau)) = \sin^2(2\theta_o)\sin^2\left[\frac{\Delta m_o^2 L}{4K}\right]$$

From this result it can be noted that the maximum probability is at $\sin^2(2\theta_o)$, where θ_o is the vacuum mixing angle. From this we can see that, in the case of small θ_o, the probability of flavour mixing cannot be very high. This is quite different for the case of interactions with matter.

For Interactions with Matter of Constant Density:

The derivations given above is equivalent to solving the Schrodinger Equation below:

(7) $$i\left(\frac{\partial}{\partial t}\nu\right)=\left[\frac{1}{2}\right]\begin{bmatrix}-\frac{\Delta m_o^2\cos(2\theta_o)}{K} & \frac{\Delta m_o^2\sin(2\theta_o)}{K}\\ \frac{\Delta m_o^2\sin(2\theta_o)}{K} & \frac{\Delta m_o^2\cos(2\theta_o)}{K}\end{bmatrix}\begin{bmatrix}\nu_e(0)\\ \nu_\mu(0)\end{bmatrix}$$

where the matrix in (7) is simply the mixing matrix in a vacuum. In matter of constant density, the neutrinos will interact as they propagate adiabatically. That interaction has the effect of adding $\sqrt{2}\,G_f N_e$ [1,3] to the {1,1} element and subtracting it from the {2,2}element of the mixing matrix. The resulting Schrodinger equation is:

(8)

$$i\left(\frac{\partial}{\partial t}\nu\right)=\left[\frac{1}{2}\right]\begin{bmatrix}-\frac{\Delta m_o^2\cos(2\theta_o)}{K}+\sqrt{2}\,G_f N_e & \frac{\Delta m_o^2\sin(2\theta_o)}{K}\\ \frac{\Delta m_o^2\sin(2\theta_o)}{K} & \frac{\Delta m_o^2\cos(2\theta_o)}{K}-\sqrt{2}\,G_f N_e\end{bmatrix}\begin{bmatrix}\nu_e(0)\\ \nu_\mu(0)\end{bmatrix}$$

where G_f is defined as the Fermi coupling constant and N_e is the number density of electrons of the matter. We now define two parameters analagous to θ_o and Δm_o^2 in (7), θ_N and Δm_N^2, for the case of interactions with matter of constant density:

(9) $$\frac{\Delta m_N^2\cos(2\theta_N)}{2K}=\frac{\Delta m_o^2\cos(2\theta_o)}{2K}-\sqrt{2}\,G_f N_e$$

(10) $$\frac{\Delta m_N^2\sin(2\theta_N)}{2K}=\frac{\Delta m_o^2\sin(2\theta_o)}{2K}$$

Then the solutions are[1]:

(11) $$2\theta_N=\operatorname{arccot}\left(\cot(2\theta_o)-\frac{K\sqrt{2}\,G_f N_e}{\Delta m_o^2\sin(2\theta_o)}\right)$$

(12) $$\Delta m_N^2=\frac{\Delta m_o^2\sin(2\theta_o)}{\sin(2\theta_N)}$$

Notice that the matter mixing parameters are defined explicitly in terms of the vacuum mixing parameters θ_o and Δm_o^2. In addition, small values for θ_o can have large effects on the magnitude of θ_N

and result in resonance. Resonance is merely a term used when the MSW Effect is the greatest. In this case, resonance occurs when $2\,\theta_N$ is a maximum, i.e. when $2\,\theta_N = \frac{\pi}{2}$. This is equivalent to solving the following equation for N_e in terms of θ_o and $\Delta\, m_o$:

$$\cot(2\,\theta_o) - \frac{\left(\frac{\Delta\, m_o^2}{K}\right)^{(-1)} \sqrt{2}\, G_f N_e}{\sin(2\,\theta_o)} = 0 \tag{13}$$

Thus, in matter, resonance occurs when:

$$N_e(res) = \frac{\Delta\, m_o^2 \cos(2\,\theta_o)}{K\sqrt{2}\, G_f} \tag{14}$$

Resonance, then, is largely dependent upon the electron number density N_e as opposed to the vacuum mixing angle θ_o; as a result the potential for neutrinos to change flavours is magnified significantly.

For Interactions with Matter Of Varying Density:

If matter has varying density, we must amend the factor we added/subtracted to the diagonal entries of the Hamiltonian in (7) [1]. Simply let N_e go to $N_e(t)$ in the Schrodinger equation in (8). The final result is:

$$(15)\quad \begin{bmatrix} \nu_1(t) \\ \nu_2(t) \end{bmatrix} =$$

$$\begin{bmatrix} C(\theta_N(t)) & -S(\theta_N(t)) \\ S(\theta_N(t)) & C(\theta_N(t)) \end{bmatrix} \begin{bmatrix} e^{\left(\frac{i\,\Delta\, m_N(t)^2\, t}{2K} + i\,K\,t\right)} & 0 \\ 0 & e^{\left(-\frac{i\,\Delta\, m_N(t)^2\, t}{2K} - i\,K\,t\right)} \end{bmatrix} \begin{bmatrix} C(\theta_N(t)) & S(\theta_N(t)) \\ -S(\theta_N(t)) & C(\theta_N(t)) \end{bmatrix} \begin{bmatrix} \nu_1(0) \\ \nu_2(0) \end{bmatrix}$$

where $C(\theta_N(t))$ represents $\cos(\theta_N(t))$ and $S(\theta_N(t))$ represents $\sin(\theta_N(t))$. Again, it is important to note the free parameters which we solved for above, namely θ_o and $\Delta\, m_o^2$, for they will be what are used to determine the solution in (15).

III. Data

The MSW Effect is important to solar neutrino oscillations in that, according to the definition of the effect, the presence of matter enhances or magnifies oscillations, as can be noted from the results given in (11). Resonance occurs when the probability of a transition is at a maximum. Much of the MSW parameter space has been excluded by means of recorded data from previous experiments [1,3,6,7,8]. As detector capabilities improve, more information can be accumulated on the nature of neutrinos. Further narrowing of the window of parameter space, which we are interested in observing as the result of chi-squared fits, will aid in constraining θ_o and Δm_o^2. As it stands, results from the detector at SNO in Canada have constrained them to: $\Delta m_o^2 = 5 \times 10^{(-5)} eV^2$; $\theta_o = 0.53$ radians [7]. Thus it is reasonable for the purposes of this paper to use the above constraints for our own calculations as well.

While on the topic of the MSW parameter space, it is appropriate again to mention that maximum transition probability will occur at resonance. The resonance length for oscillations [5] in the vacuum is determined when the probability in the vacuum is at a maximum, i.e. when (6) is a maximum, or when:

$$L_{res} \sim 1.24 \left(\frac{\Delta m_o^2}{K} \right)^{-1} \qquad (16)$$

where, as previously stated, $L = c\, t$ for relativistic particles.

While our predictions for enhancement of oscillations due to the solar eclipse effect are very promising, there is a slight problem to be overcome in collecting data. Currently the Sudbury Neutrino Observatory only detects 10 neutrino events per day [1]. Eclipses do not last more than a couple of hours. Thus, while the information that can be attained utilizing the MSW Effect may be significant, the duration of an eclipse is such that it will hamper the pace of progress. More on this topic will be discussed below.

IV. Astronomy

The importance of the percent coverage of the sun during a solar eclipse was mentioned in Section I. It is appropriate that we derive a formula to determine this value defined as C. C will be dependent upon three parameters[3]:

$$\text{Ratio of Apparent Radii: } a = \frac{r_m}{r_s}$$

$$\text{Duration of Eclipse: } T$$

$$\text{Impact Parameter: } p = \frac{l}{r_m}$$

While the percent coverage C can be defined in terms of time t, it is convenient to define it instead in terms of the apparent separation between the centers of the sun and the moon, d, where d is actually d(t):

(16) $$d(t) := \sqrt{p\left(1 - \left[2\frac{t}{T}\right]^2\right) + (1+a)^2\left[2\frac{t}{T}\right]^2}$$

There are four distinct cases [3] to analyze:

Case 1: $1 + a \le d$

then $C(d) = 0$

Case 2: $(d < 1 + a)$ and $(\sqrt{|1 - a^2|} \le d)$

then $C(d) = f(u) + a^2 f\left(\frac{u}{a}\right)$ where $f(u) = \left[\frac{1}{\pi}\right](\arcsin(u) - u\sqrt{1 - u^2})$ (17)

and $u = \left[\frac{1}{2d}\right]\sqrt{([(1+a)^2 - d^2][d^2 - (1-a)^2])}$ (18)

Case 3: $\{d < \sqrt{|1-a^2|}\}$ and $\{|1-a| \le d\}$

then subcase a) for $a \le 1$ $\quad C(d) = f(u) + a^2\left[1 - f\left(\frac{u}{a}\right)\right]$

subcase b) for $1 < a$ $\quad C(d) = 1 - f(u) + a^2 f\left(\frac{u}{a}\right)$

Case 4: $\{d < |1-a|\}$ and $\{0 \le d\}$

then subcase a) for $1 \le a$ $\quad C(d) = 1$

subcase b) for $1 < a$ $\quad C(d) = a^2$

Recall that $a = \frac{r_m}{r_s}$; this can be approximated to give $a = 1$ since the observed radii of the sun and moon are approximately equivalent. As a result, only Case 2 is of any importance to us, i.e. when $0 \le d < 2$. [In the first case, $2 < d$ is equivalent to there being no eclipse at all; for Case 3 and 4, we have $0 < d$ and $d < 0$, which are also irrelevant.]

Then:

$$C(d) = 2\,f(u) \text{ where } f(u) = \frac{\arcsin(u) - u\sqrt{1-u^2}}{\pi},\ u = \frac{\sqrt{4-d^2}}{2}, \text{ and } d = \sqrt{p + \left[\frac{4\,t^2}{T^2}\right](4-p)}\ .$$

Now, for convenience we will introduce a useful definition: let $x = \frac{d_M}{2\,R_M}$ where d_M is the distance a given neutrino travels through the moon on its way to the earth and where R_M is the radius of the moon. If x=1, then the neutrino travels directly from the sun's core and through an entire diameter of the moon which is equivalent to the occurence of a total solar eclipse so that the resulting coverage, C, is 100%. If x = 0, then the neutrino does not pass through the moon at all, in which case there can be no solar eclipse effect. That is, while the resulting coverage of the moon may be greater than 0%, it is also less than 39%; thus the moon will never cross in front of the center of the sun. Since x is defined in terms of d_M, this ratio can also be defined in terms of the coverage, C, in accordance with the two formulas below [4]:

(19) $$x=\sqrt{4z[2-z]-3}$$

(20) $$C=\left[\frac{2}{\pi}\right]\left(\cos(1-z)^{-1}-(1-z)\sqrt{z(2-z)}\right)$$

Yet one more distance of import to us is the distance which solar neutrinos travel through the earth (i.e. during a 'double eclipse'). The point at which the neutrino makes its initial contact with Earth's surface is correlated in the following manner:

for initial points of contact (α, β), where α is the latitude and β is the longitude, and

for the secondary points (λ, σ), where λ is the latitude and σ is the longitude, and (λ, σ) are the coordinates of the double eclipse),

we have[3]:

(21) $$\lambda=\alpha+2\delta$$

(22) $$\sigma=\pi-2\Theta_{UT}-\beta$$

(23) $$\sin(\delta)=\sin\left(\frac{23.5\pi}{180}\right)\sin\left(\frac{2\pi t}{T_Y}\right)$$

where the zero of time t is set at midnight of the autumnal equinox, Θ_{UT} is the angle which corresponds to the universal time, and T_Y is the length of a year. It follows that the path the neutrino travels along inside the earth is simply the path from (α, β) to (λ, σ), which is just a chord of the outer spherical shell of the earth. Hence we can define this path length in terms of time t:

(24) $$d_E=2R_E\left(\sin(\lambda)\sin(\delta)+\cos(\lambda)\cos(\delta)\cos\left(\frac{2\pi t}{T_D}\right)\right)$$

where R_E is the radius of the earth and T_D is the length of a day.

We now know how to determine the percent coverage of the sun during a solar eclipse as well as how to determine the distance that a neutrino will travel through each of two mediums (the moon and the earth) given the coverage, C. The impact parameters for the eclipses of percent coverage greater than 39% at each detector, as well as the values for x (defined above) have been calculated and are given in Tables 1 and 2 below:

Table 1: The impact parameters, $p = \frac{l}{r_m}$, given for each eclipse with significant magnitude (i.e. greater than 0.39) at each detector site.

		Impact Parameters			
Date		Detector	Sites		
	Gran Sasso	Homestake	Kamioka	Sage	SNO
5/31/2003	0.405405	0.8		0.736842	0.63158
10/14/2004		0.9			
10/3/2005	0.6153846	0.08108			0.02597
3/29/2006	0.83721		0.52381	0	
3/19/2007	0.83333			0.52381	
8/1/2008		0.626506	0.240964	1.2857*	0.963855
7/22/2009			0.511628		
1/15/2010		0.083333	0.378378		0.324324
1/4/2011	0.46154			0.962025	
6/1/2011				1.14286*	
5/21/2012		0.586667	0.32		0.93333
11/3/2013		0.8780488	0.51613		0.619048
10/23/2014		0.846154			0.78481
3/20/2015	0.65116	0.4285714			0.2857
3/8-9/2016		0.714286			1
8/21/2017		0.0487805	0.487805		0.571429
1/6/2019			1.1579*		
12/26/2019	0.179487	0.128205			0.10256
6/21/2020			1.06173*		
6/10/2021		0.162162	0.88		0.162162
10/25/2022				0.609756	
10/14-15/2023		1	0.756756		
4/8/2024	0.07143	0.744186			0.232558
3/29/2025		0.5238095			
8/12/2026	0.095238			0.119048	
8/2/2027	0.511628			1.209302*	
1/26/2028			0.109589	0.32	
1/14/2029		0.410256			0.455696
6/1/2030	0.586667		0.324324	0.106667	
11/14-15/2031	0.48889		0.246914	0.390244	
11/3/2032			0.925		
3/30/2033		1.1628*	0.674699		
3/20/2034			0.52381	0.873563	
9/2/2035			0	0.878049	

*for Impact Parameter > 1, the eclipse will not cover the center of the sun

Table 2: The values for $x = \frac{d_M}{2\,R_M}$, where d_M is the distance the neutrino will travel through the moon and R_M is the radius of the moon, for each eclipse of significance at each detector site.

		x Values			
Date		Detector	Sites		
	Gran Sasso	Homestake	Kamioka	Sage	SNO
5/31/2003	0.6120628	0.446564		0.7952188	0.768888
10/14/2004		0.446564			
10/3/2005	0.87062131	0.98756			0.98756
3/29/2006	0.7547733		0.9185573	0.996909	
3/19/2007	0.446564			0.9185573	
8/1/2008		0.8795725	0.996909	**	0.446564
7/22/2009			0.8881214		
1/15/2010		0.9875601	0.8795725		0.948935
1/4/2011	0.8795725			0.8612526	
6/1/2011				0.179688	
5/21/2012		0.9114886	0.995545		0.3674171
11/3/2013		0.948935	0.948935		0.446564
10/23/2014		0.7952188			0.6120628
3/20/2015	0.7952188	0.948935			0.948935
3/8-9/2016		0.690927			0.179688
8/21/2017		0.995545	0.9185573		0.8795725
1/6/2019			0.3674171		
12/26/2019	0.948935	0.996909			0.996909
6/21/2020			0.4093907		
6/10/2021		0.996909	0.5892572		0.9849212
10/25/2022				0.8514497	
10/14-15/2023		0.8881214	0.8795725		
4/8/2024	0.996909	0.8074169			0.9849212
3/29/2025		0.9185573			
8/12/2026	0.8612526			0.996909	
8/2/2027	0.9185573			**	
1/26/2028			0.98756	0.9717119	
1/14/2029		0.9252832			0.8411945
6/1/2030	0.948935		0.9185573	0.9788539	
11/14-15/2031	0.8795725		0.98756	0.948935	
11/3/2032			0.6120628		
3/30/2033		0.3674171	0.8304669		
3/20/2034			0.9185573	0.6537164	
9/2/2035			0.99950636	0.446564	

V. The Detectors

Currently there are numerous detectors capable of gathering data on solar neutrino events. They are: Borexino, Dumand, Gallex, GNO (Gran Sasso Neutrino Observatory), Homestake, Icarus, Sage, SNO, and Super-Kamiokande [4]. Hellaz and Heron have been proposed, but no location has been decided upon. In addition, Hyper-K (in Kamioka, Japan), UNO (Underground Neutrino Observatory; in affiliation with Homestake in South Dakota), and NUSL (National Underground Science and engineering Laboratory at WIPP in Carlsbad, New Mexico) are each proposed laboratories which will include neutrino observatories of mcuh greater scale than any presently in use. Hyper-K is supposed to have up to 50 times the volume as Super-K. At present, decisions are being made as to whether or not an American National Laboratory should be constructed at Homestake (which would be UNO) or at WIPP (Waste Isolation Pilot Plant) in Carlsbad, NM. Unfortunately, Dumand (in Hawaii) was shut down in 1996 due to lack of funding so this detector is not currently relevant to furthering progress with respect to the solar neutrino problem. In addition, Super-K experienced immense damage to more than 10,000 phototubes within their detector last year. While it has only recently started running again, it is not at full capacity. The Borexino detector is, along with Gallex, GNO, and Icarus, affiliatied with the Gran Sasso National Laboratory in Milan. It is currently under construction and will use high purity water to insulate photomultiplier tubes to try to detect solar neutrinos via inelastic neutrino-electron scattering. GNO is the successor of Gallex which uses gallium detectors. Solar neutrinos interact with gallium to become germanium. Homestake is located in the depths of an old gold mine in North Dakota. The first neutrino detector was built there circa 1970 and used carbon tetrachloride as its solution. Solar electron neutrinos interacted with the chlorine to form argon, and measurements of solar neutrino events were calculated based on the amount of argon present in the solution at various intervals. There is now another detector at Homestake which utilizes iodine as its active ingredient. Sage is a detector which also uses gallium. It is located at the Baksum Neutrino Observatory in N. Caucasus, Russia. SNO is located in a Creighton nickel mine near Sudbury, Ontario. It utilizes heavy water as well and is responsible for definitive evidence of neutrino oscillations. Super-Kamiokande is located in Kamioka, Japan, and used photomultiplier tubes and high purity water.

Perhaps the most important and most relevant data needed in regards to the detectors at present, however, are their latitudinal and longitudinal positions. Recall that we are interested in when the solar eclipse effect may be observed. In order to know, we must determine when a solar eclipse will be 'visible' at each detector site, where 'visible' may refer to observable to the naked eye as well as through a transparent earth (i.e. in the situation of a double eclipse). Below are listed the latitude/longitude for each site:

Table 3: Latitude and longitude positions of major solar neutrino detectors.

Site	Latitude	Longitude	Notes
Carlsbad, NM	32.4°N	104.28°W	in the United States
Gran Sasso	42.48°N	13.5°E	includes Borexino, Gallex, GNO, and Icarus
Homestake	44.4°N	103.7°W	includes UNO
Kamioka, Japan	36.4°N	140.0°E	includes Kamiokande, Super-K, and Hyper-K
Sage	43.68°N	43.53°E	in Russia
SNO	46.48°N	81.0°W	in Canada

Given the above information, we can use the computer program 'DANCE of the Planets' [9] (geared towards astronomical computations) to determine when eclipses will be 'visible' at each site through the year 2035. In addition, DANCE can calculate the maximum magnitude, or coverage C, of the eclipses. This data is listed in the following Tables 4-9. Knowing this information, we can now go on to compute the expected drop in neutrino events at each detector. This will be done by using the theory detailed in Section IV.

Table 4: Eclipse data for the detector at Carlsbad, NM, including (1) the date of the eclipse, (2) whether or not the eclipse is observable from the detector site, (3) whether or not the eclipse is visible, as through a transparent earth, from the detecor site, (4) the maximum percent coverage (or magnitude) during the eclipse, (5) the duration (in minutes) of the eclipse, and (6) the time the eclipse will begin.

Date	Observable	Visible	Magnitude	Duration (min)	Start Time
10/3/2005	N	Y	~0.70	83	1:36
8/1/2008	N	Y	~0.40	72	1:30
1/15/2010	N	Y	~0.50	89	23:38
5/21/2012	Y	Y	0.92	124	17:32
11/3/2013	N	Y	~0.90	97	3:27
10/23/2014	Y	Y	0.41	108	16:02
3/20/2015	N	Y	~0.40	70	1:43
3/8-9/2016	N	Y	1	91	19:34
8/21/2017	Y	Y	0.71	237	8:27
12/26/2019	N	Y	~0.55	82	22:01
6/10/2021	N	Y	~0.80	96	2:02
10/14-15/2023	Y	Y	0.94	177	8:18
4/8/2024	Y	Y	0.87	152	10:14
3/29/2025	N	Y	~0.45	81	2:39
6/1/2030	N	Y	~0.40	83	22:56

Table 5: Eclipse data for the detector at Gran Sasso, including (1) the date of the eclipse, (2) whether or not the eclipse is observable from the detector site, (3) whether or not the eclipse is visible, as through a transparent earth, from the detecor site, (4) the maximum percent coverage (or magnitude) during the eclipse, (5) the duration (in minutes) of the eclipse, and (6) the time the eclipse will begin.

Date	Observable	Visible	Magnitude	Duration (min)	Start Time
5/31/2003	Y	Y	0.51	107	3:22
10/3/2005	Y	Y	0.69	163	8:58
3/29/2006	Y	Y	0.59	136	10:30
3/19/2007	N	Y	~0.45	82	2:38
1/4/2011	Y	Y	0.7	163	7:56
3/20/2015	Y	Y	0.62	138	9:26
12/26/2019	N	Y	~0.80	108	4:02
4/8/2024	N	Y	~0.95	93	19:55
8/12/2026	Y	Y	0.67	100	18:31
8/2/2027	Y	Y	0.75	142	9:04
6/1/2030	Y	Y	0.8	129	5:03
11/14/2031	N	Y	~0.70	83	21:44

Table 6: Eclipse data for the detector at Homestake, including (1) the date of the eclipse, (2) whether or not the eclipse is observable from the detector site, (3) whether or not the eclipse is visible, as through a transparent earth, from the detecor site, (4) the maximum percent coverage (or magnitude) during the eclipse, (5) the duration (in minutes) of the eclipse, and (6) the time the eclipse will begin.

Date	Observable	Visible	Magnitude	Duration (min)	Start Time
5/31/2003	N	Y	~0.45	91	21:24
10/14/2004	N	Y	~0.45	91	19:31
10/3/2005	N	Y	~0.90	101	1:22
8/1/2008	N	Y	~0.70	86	1:26
1/15/2010	N	Y	~0.90	99	0:09
5/20/2012	Y	Y	0.74	123	17:18
11/3/2013	N	Y	~0.80	98	3:33
10/23/2014	Y	Y	0.62	146	14:12
3/20/2015	N	Y	~0.80	88	2:32
3/9/2016	N	Y	~0.55	86	19:32
8/21/2017	Y	Y	0.94	161	9:29
12/26/2019	N	Y	~0.95	94	21:51
6/10/2021	Y	Y	~0.95	109	2:53
10/14/2023	Y	Y	0.71	162	8:18
4/8/2024	Y	Y	0.63	135	10:40
3/29/2025	N	Y	~0.75	90	2:48
1/14/2029	Y	Y	0.76	152	7:57
3/30/2033	Y	Y	0.43	120	9:59

Table 7: Eclipse data for the detector at Kamioka, including (1) the date of the eclipse, (2) whether or not the eclipse is observable from the detector site, (3) whether or not the eclipse is visible, as through a transparent earth, from the detecor site, (4) the maximum percent coverage (or magnitude) during the eclipse, (5) the duration (in minutes) of the eclipse, and (6) the time the eclipse will begin.

Date	Observable	Visible	Magnitude	Duration (min)	Start Time
3/29/2006	N	Y	~0.75	88	19:52
8/1/2008	N	Y	~0.95	97	20:04
7/22/2009	Y	Y	0.71	128	11:35
1/15/2010	N	Y	~0.70	113	17:51
5/20/2012	Y	Y	0.94	150	6:31
11/3/2013	N	Y	~0.80	83	21:24
8/21/2017	N	Y	~0.75	86	1:32
1/6/2019	Y	Y	0.43	171	8:45
6/21/2020	Y	Y	0.44	110	16:12
6/10/2021	N	Y	~0.50	93	20:58
10/15/2023	N	Y	~0.70	164	0:57
1/26/2028	N	Y	~0.90	98	0:07
6/1/2030	Y	Y	0.8	137	15:53
11/15/2031	N	Y	~0.90	105	3:41
11/3/2032	Y	Y	0.51	141	14:19
3/30/2033	N	Y	~0.65	83	2:37
3/20/2034	N	Y	~0.75	85	20:39
9/2/2035	Y	Y	0.98	168	8:46

Table 8: Eclipse data for the detector at Sage, including (1) the date of the eclipse, (2) whether or not the eclipse is observable from the detector site, (3) whether or not the eclipse is visible, as through a transparent earth, from the detecor site, (4) the maximum percent coverage (or magnitude) during the eclipse, (5) the duration (in minutes) of the eclipse, and (6) the time the eclipse will begin.

Date	Observable	Visible	Magnitude	Duration (min)	Start Time
5/31/2003	Y	Y	0.62	115	5:05
3/29/2006	Y	Y	1.05	144	13:05
3/19/2007	N	Y	~0.75	95	4:18
8/1/2008	Y	Y	0.39	122	12:34
1/4/2011	Y	Y	0.68	180	10:37
6/1/2011	N	Y	~0.40	77	23:37
10/25/2022	Y	Y	0.67	151	12:45
8/12/2026	N	Y	~0.95	90	20:16
8/2/2027	Y	Y	0.39	112	12:00
1/26/2028	N	Y	~0.85	114	18:46
6/1/2030	Y	Y	0.87	163	7:02
11/14/2031	N	Y	~0.80	86	23:13
3/20/2034	Y	Y	0.53	131	13:12
9/2/2035	N	Y	~0.45	84	2:04

Table 9: Eclipse data for the detector at SNO, including (1) the date of the eclipse, (2) whether or not the eclipse is observable from the detector site, (3) whether or not the eclipse is visible, as through a transparent earth, from the detecor site, (4) the maximum percent coverage (or magnitude) during the eclipse, (5) the duration (in minutes) of the eclipse, and (6) the time the eclipse will begin.

Date	Observable	Visible	Magnitude	Duration (min)	Start Time
5/31/2003	N	Y	~0.60	95	22:14
10/3/2005	N	Y	~0.90	106	3:03
8/1/2008	N	Y	~0.45	86	3:11
1/15/2010	N	Y	~0.80	100	1:04
5/21/2012	Y	Y	0.43	103	19:18
11/3/2013	N	Y	~0.45	105	5:25
10/23/2014	Y	Y	0.51	126	16:31
3/20/2015	N	Y	~0.80	92	3:27
3/8/2016	N	Y	~0.40	76	21:21
8/21/2017	Y	Y	0.7	150	12:09
12/26/2019	N	Y	~0.95	92	23:30
6/10/2021	Y	Y	0.89	111	3:50
4/8/2024	Y	Y	0.89	142	13:06
1/14/2029	Y	Y	0.66	177	9:47

VI. Calculations

As more data is gathered with respect to solar neutrino events at detectors throughout the world , the MSW parameters become more accurate. Neutrino observations depend on the energy range of these detectors, which in turn are determined by the active material. Below are solar neutrino detectors' active materials and their allowed energy ranges [10]:

Table 10: Active materials and energy ranges (in MeV) for major detector sites.

Detector	Active Material	Energy Range (MeV)
Gran Sasso	Gallium	1.02-15
Homestake	Chlorine	0.8-20
Kamioka	Deuterium	7.3-20
Sage	Gallium	0.233-8
SNO	Chlorine	0.8-20

Now we can use these parameters in order to determine the probability that an electron neutrino detectable at each site will a) transition (to another flavour) in the sun, and b) transition in the moon. The probability for transition (or oscillation) to occur in the sun is dependent upon the energy of the neutrino which in turn may result in resonance. Recall the condition for resonance in matter is:

(25)
$$N_e(res) = \frac{\Delta m_o^2 \cos(2\,\theta_o)}{K\sqrt{2}\,G_f}$$

Thus, for each of the sets of parameters above, resonance will be reached. Therefore, the probability for transition in the sun, P(ts), is determined by [3]:

(26)
$$\mathrm{P}(ts) := \sin^2(2\,\theta_o) + P_x \cos(2\,\theta_o)$$

where:

(27)
$$P_x = \mathrm{e}^{\left(\frac{\left(-\frac{\pi}{2}\right)\sin(2\,\theta_o)^2\,\Delta m_o^2}{\cos(2\,\theta_o)\,2\,K\left[\frac{1}{N_{res}}\right]}\right)}$$

P(ts) for each detector site has been evaluated and is given below:

$$P(ts) = .3787$$

In order to determine the probability for oscillations in the moon, we will also need the impact parameter p (mentioned previously in Section IV) as well as the distance travelled through the moon $2\,R_m\,x$ where R_m is the radius of the moon and x is as described in Section IV as well. We can calculate this probability using the following formula [1]:

$$P(tm) = \sin^2(2\,\theta_o)\sin^2\left[\frac{4320\,\Delta\,m_N^{\;2}\sqrt{1 - p^2 - \left(\frac{t}{R_m}\right)^2}}{K}\right] \tag{28}$$

This has been done for three values of $\Delta\,m_o^{\;2}$: 1) the best fit parameter (given in Section III)

2) (best fit parameter)*(1/3)

3) (best fit parameter)*(3)

These calculations are listed in Tables 11-13 below. MAPLE was used for the computations, and the appropriate code is included in Figure 1 in the Appendix.

Table 11: Probability of transition in the moon given $\Delta m_o^2 = 1.67\ 10^{(-5)} eV^2$ and $\theta_o = 0.53$ radians for each eclipse of significance at the appropriate detector sites.

	Probability of Transition in the Moon				
Date		Detector Sites			
	Gran Sasso	Homestake	Kamioka	Sage	SNO
5/31/2003	0.0005897	-0.00187		0.0000155	0.00287
10/14/2004		-0.00187			
10/3/2005	-0.00383	-0.00112			-0.00112
3/29/2006	0.00248		0.00541	0.000712	
3/19/2007	-0.00136			0.00331	
8/1/2008		-0.000689	0.00116	**	-0.00187
7/22/2009			-0.00305		
1/15/2010		-0.00112	-0.000456		0.00296
1/4/2011	-0.000502			-0.00138	
6/1/2011				-0.000319	
5/21/2012		-0.00238	0.00411		-0.00385
11/3/2013		0.00296	0.001955		-0.00187
10/23/2014		0.0000384			0.0008099
3/20/2015	0.00002796	0.00296			0.00296
3/8-9/2016		0.00519			-0.000788
8/21/2017		0.00622	0.00541		-0.000689
1/6/2019			-0.00255		
12/26/2019	0.00215	0.00176			0.00176
6/21/2020			-0.000348		
6/10/2021		0.00176	-0.00427		0.004699
10/25/2022				-0.00184	
10/14-15/2023		-0.00461	-0.00456		
4/8/2024	0.00128	0.00525			0.004699
3/29/2025		0.00817			
8/12/2026	-0.00248			0.000712	
8/2/2027	0.00595			**	
1/26/2028			-0.0007403	0.000716	
1/14/2029		0.001603			-0.00178
6/1/2030	0.00215		0.00541	-0.000487	
11/14-15/2031	-0.000502		-0.0007403	0.001196	
11/3/2032			0.000536		
3/30/2033		-0.00385	0.0002977		
3/20/2034			0.00541	0.00156	
9/2/2035			0.00129	-0.000757	

Table 12: Probability of transition in the moon given $\Delta m_o^2 = 5\ 10^{(-5)} eV^2$ and $\theta_o = 0.53$ radians for each eclipse of significance at the appropriate detector sites.

	Probability of Transition in the Moon				
Date		Detector Sites			
	Gran Sasso	Homestake	Kamioka	Sage	SNO
5/31/2003	0.00251	0.000901		0.000313	-0.00379
10/14/2004		0.000901			
10/3/2005	-0.00315	0.00135			0.00135
3/29/2006	0.00799		0.00577	-0.000377	
3/19/2007	0.000656			0.00353	
8/1/2008		0.00165	-0.000616	**	0.000901
7/22/2009			-0.00412		
1/15/2010		0.00135	0.00109		0.00562
1/4/2011	0.001201			0.00118	
6/1/2011				0.0000584	
5/21/2012		-0.00326	0.000865		-0.00113
11/3/2013		0.00562	0.00372		0.000901
10/23/2014		0.000773			0.00345
3/20/2015	0.000563	0.00562			0.00562
3/8-9/2016		0.00243			0.000144
8/21/2017		0.00131	0.00576		0.00165
1/6/2019			-0.000749		
12/26/2019	0.00409	-0.000931			-0.000931
6/21/2020			-0.000167		
6/10/2021		-0.000931	-0.00146		0.00336
10/25/2022				-0.000197	
10/14-15/2023		-0.00623	0.00109		
4/8/2024	-0.000678	0.00134			0.00336
3/29/2025		0.00872			
8/12/2026	0.00212			-0.000377	
8/2/2027	0.00635			**	
1/26/2028			0.000891	0.000218	
1/14/2029		-0.00438			-0.000556
6/1/2030	0.00409		0.00577	0.0000208	
11/14-15/2031	0.001201		0.000891	0.00227	
11/3/2032			0.00228		
3/30/2033		-0.00113	-0.00459		
3/20/2034			0.00577	0.00228	
9/2/2035			0.00358	0.000364	

Table 13: Probability of transition in the moon given $\Delta m_o^2 = 1.5\ 10^{(-4)} eV^2$ and $\theta_o = 0.53$ radians for each eclipse of significance at the appropriate detector sites.

	Probability of Transition in the Moon				
Date		Detector Sites			
	Gran Sasso	Homestake	Kamioka	Sage	SNO
5/31/2003	0.00356	0.00581		0.0004201	-0.00404
10/14/2004		0.00581			
10/3/2005	0.00383	0.00833			0.00833
3/29/2006	-0.00147		0.00216	0.000826	
3/19/2007	0.00423			0.00132	
8/1/2008		0.00172	0.00135	**	0.00581
7/22/2009			-0.004999		
1/15/2010		0.00833	0.00114		-0.00796
1/4/2011	0.00126			-0.00142	
6/1/2011				0.0000953	
5/21/2012		-0.00364	0.00123		-0.0000648
11/3/2013		-0.00796	-0.00527		0.00581
10/23/2014		0.00104			0.00489
3/20/2015	0.000755	-0.00796			-0.00796
3/8-9/2016		-0.00189			0.000236
8/21/2017		0.00186	0.00216		0.00172
1/6/2019			-0.0000428		
12/26/2019	-0.005798	0.00204			0.00204
6/21/2020			-0.000407		
6/10/2021		0.00204	-0.00144		0.0000298
10/25/2022				-0.000897	
10/14-15/2023		-0.00756	0.00114		
4/8/2024	0.00149	-0.002496			0.0000298
3/29/2025		0.00327			
8/12/2026	-0.00256			0.000826	
8/2/2027	0.00238			**	
1/26/2028			0.00551	0.00242	
1/14/2029		0.00284			-0.0000011
6/1/2030	-0.005798		0.00216	-0.00224	
11/14-15/2031	0.00126		0.00551	-0.00322	
11/3/2032			0.00324		
3/30/2033		-0.0000648	0.000461		
3/20/2034			0.00216	-0.00246	
9/2/2035			-0.00314	0.00235	

With respect to the calculations which have been done there are a few things that must be said. The first is that these calculations show that in actuality, the observable effect of the moon on solar neutrino oscillations at our detector sites is very small (i.e. on the order of one tenth of a percent). Second, Approximations have been made; these, along with the resulting error, will be accounted for in the Conclusion. Third, the purpose of these calculations is to serve as a guide, both with respect to what to look for and as to how to approach predicting probabilities to observe.

VII. Conclusion

As more data is accumulated, it may be possible to set an upper limit on the sum of the known neutrino masses, providing for a better understanding of the Universe at the most microscopic level. Hence the reason that my particular project is of practicality. The times and dates of both single and double eclipses relevant to each of the operating and to-be-operating detectors have been determined. Given the size of each detector as well as the duration of each eclipse, predictions of the observations at these detectors have been made. These calculations span the next 32 years, thus proposed detectors such as UNO, NUSL, and Hyper-K have been explored.

One of the most important and useful pieces of information to gather from this report are times when detectors should try to examine the MSW Effect due to the moon on observed solar neutrino flux. The number of events per hour observed at any given detector is of very low order. As a result, groups working at different detectors should take advantage of the timeline given for the following reason: whenever a solar eclipse occurs with magnitude greater than 39% at more than one detector, there is a chance to increase the data collected during that eclipse. Not only that, but flux observed during the eclipse at various detectors can then be compared to verify current theory.

Examining the predicted probabilities for transitions occurring in the moon (found in Tables 11-13), there are some conclusions to be made. The probabilities calculated dependent upon the best fit parameters (Table 12) as well as those a factor of three greater (Table 13) and lesser (Table 11) are on the order of one tenth of a percent which is extremely small. One would be forced to conclude that the moon really doesn't have an effect, or at least an observable one, on neutrino oscillations. As one would expect, for $\Delta m_o^2 = (1/3)(5 \text{ x } 10^{(-5)} eV^2)$, the transition probabilities are smaller than those for $\Delta m_o^2 = (5 \text{ x } 10^{(-5)} eV^2)$. This makes sense since the transitions (while they are not very apparent) are attributed to the MSW Effect (i.e. to both the mass of the neutrinos as well as the mass through which they travel); as Δm_o^2 decreases, so too does the likelihood of a transition. Thus, it is also true and consistent that, for $\Delta m_o^2 = (3)(5 \text{ x } 10^{(-5)} eV^2)$, probabilities rise.

Correlations between the impact parameters and x (recall, $x = \frac{d_m}{2 R_m}$) and transition probabilities, however, are a bit difficult. There are some possible reasons for this. Unfortunately DANCE [9] does not possess the capability to determine distances of closest approach for centers of the sun and the moon during eclipses. They were calculated simply by my own measurements fo what appeared on the computer screen. In addition, the determination of maximum magnitude for double eclipses was also not provided by DANCE. These figures were computed by myself with estimations made to the nearest five or ten percent. As a result, it is very crucial that I reiterate that these results are not entirely accurate. Those which provide the most accurate results would be data given for single eclipses, i.e. those with a 'Y' in the 'Observable' column in Tables 4-9. Regardless, there is still much to be gained from this paper, including 32 years' worth of eclipse information which should help in preparation at detector sites for decades to come. In addition, the derivations detailed in Section II, while basic, are quite readily adaptable as knowledge with respect to neutrinos and their properties grows.

To conclude, the most can be gained from this paper by analyzing it qualitatively. Hopefully it will prove exploitable in the future.

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