

Graphene: massless electrons in the ultimate flatland

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Work supported by:

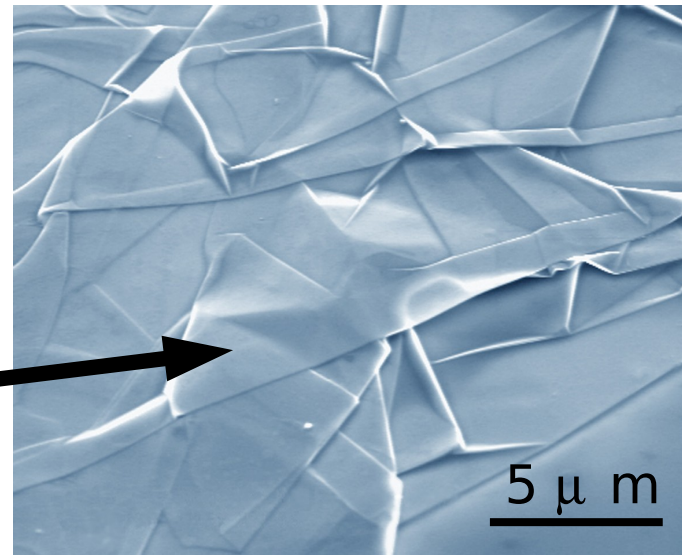
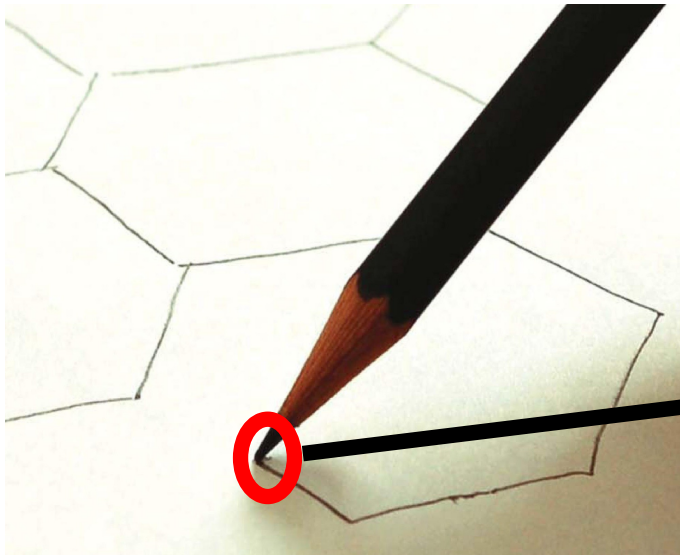
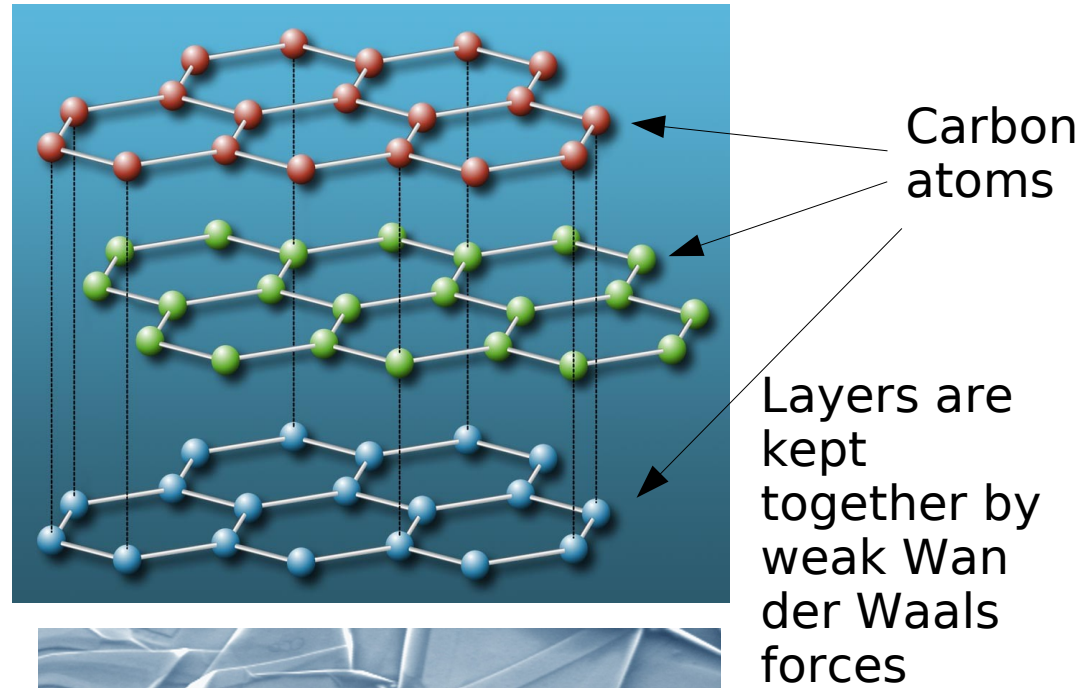


NSF-NRI

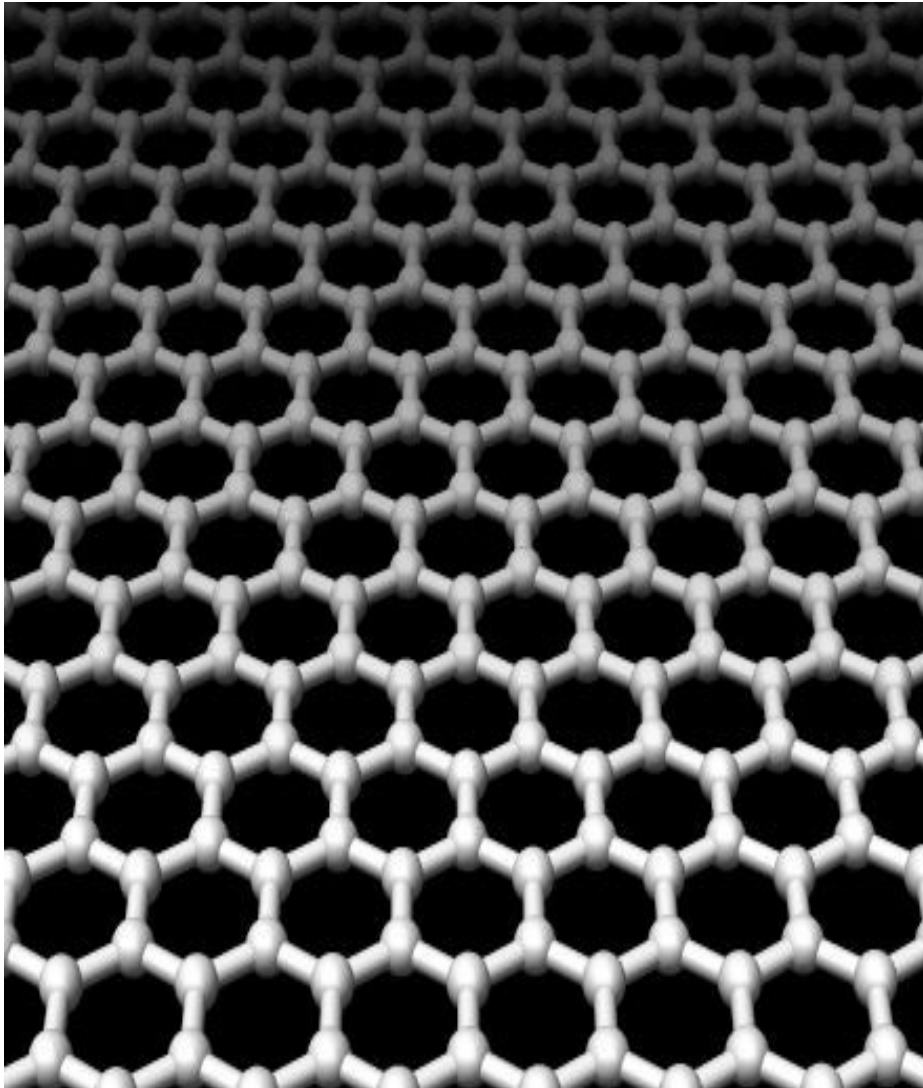


What is graphene ?

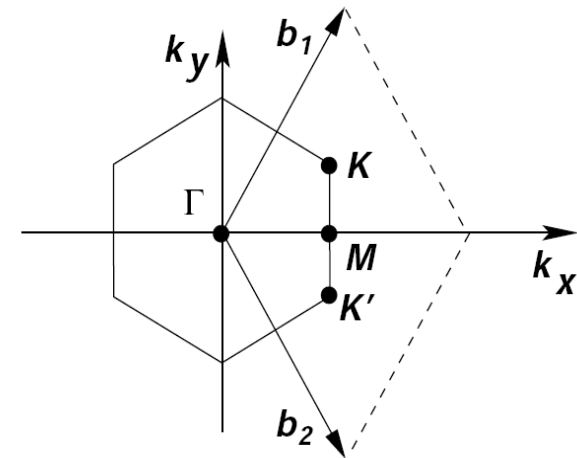
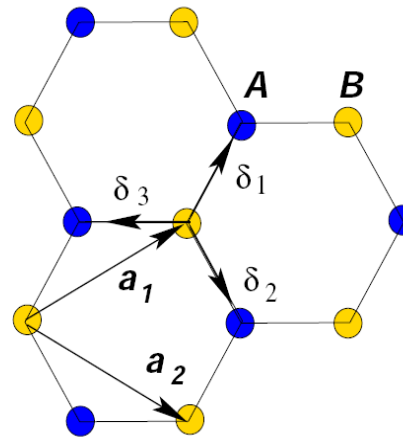
From Graphite to graphene



Graphene

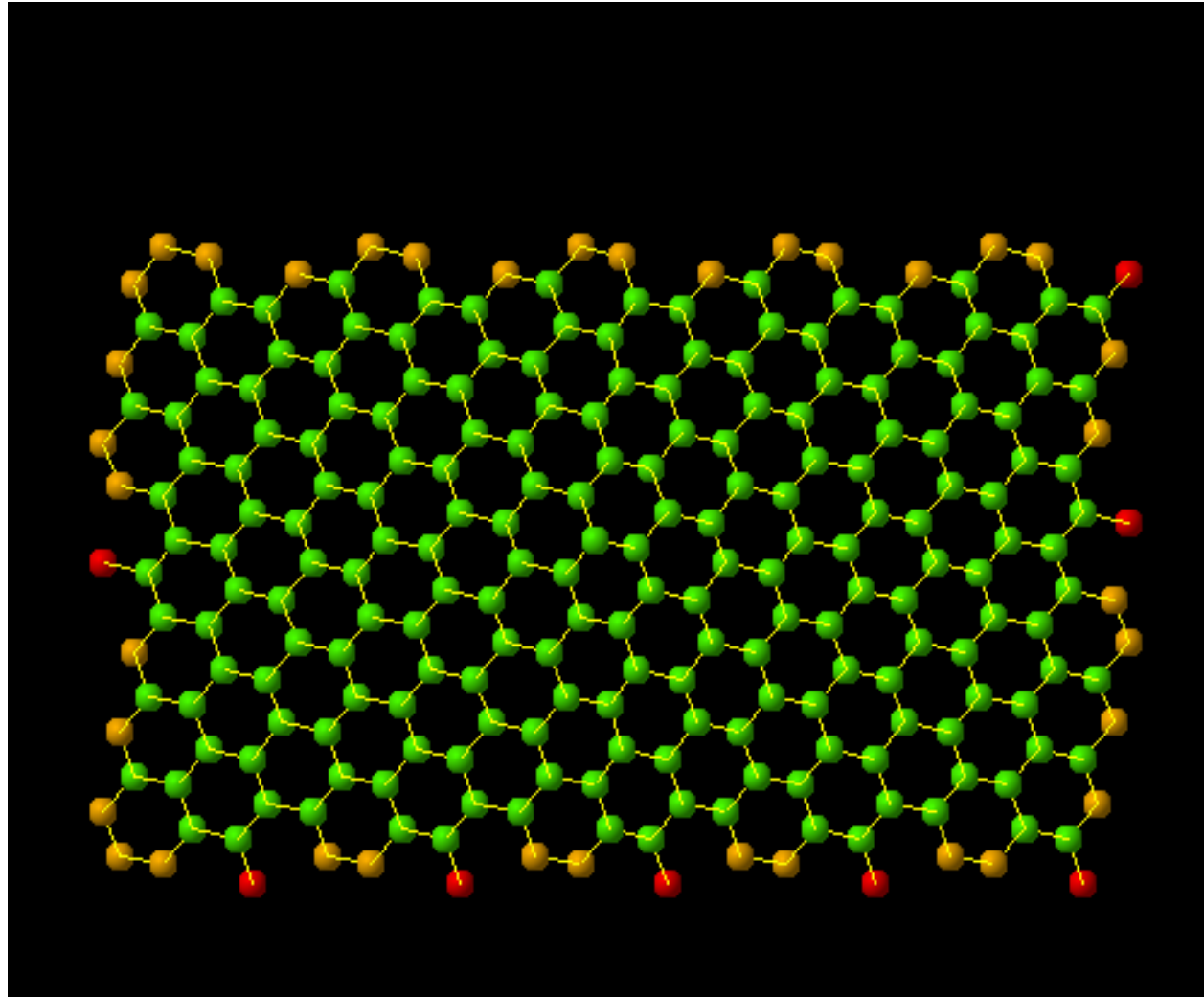


One atom thick layer of carbon atoms arranged in honeycomb structure.



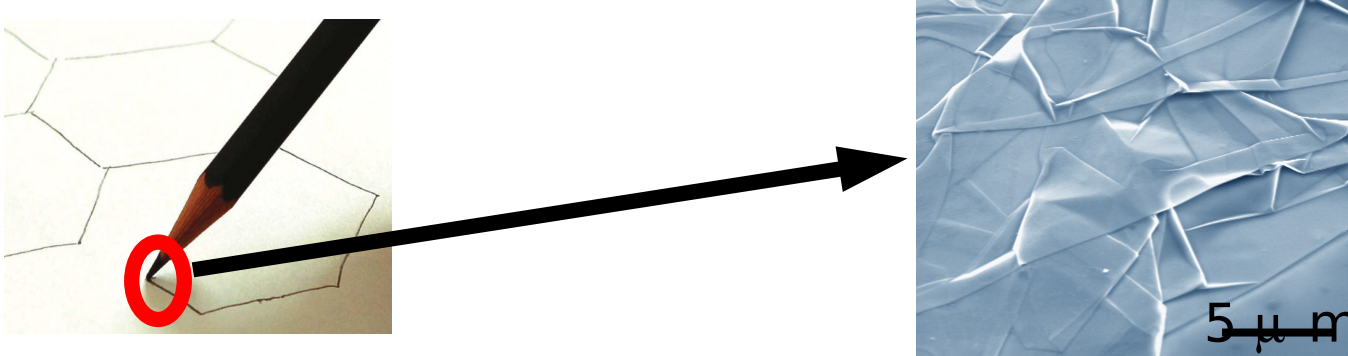
Triangular Bravais lattice with a basis. Lattice degeneracy key element to explain many of the properties of graphene.

Graphene as an unrolled nanotube



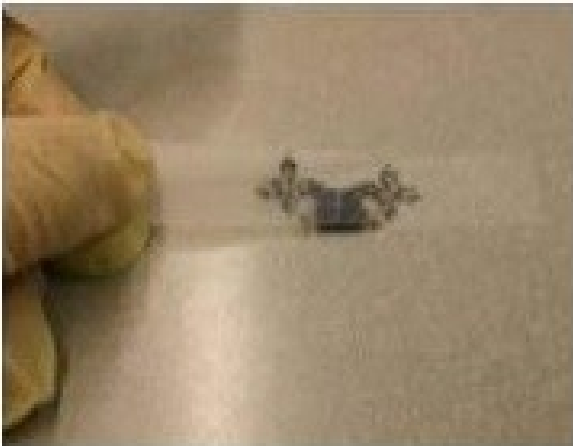
A brief history

- 1564: “Lead pencil” based on graphite was invented



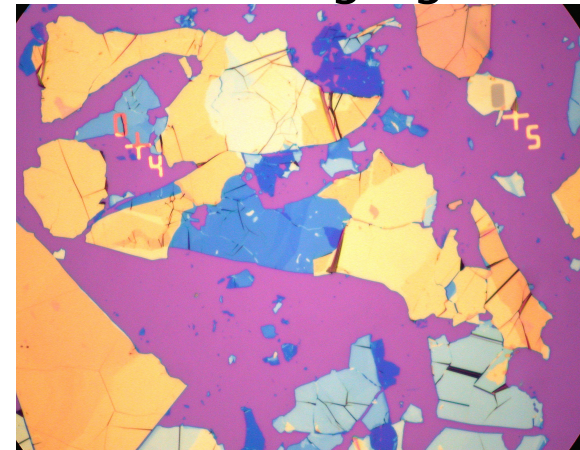
- 1946 P. R. Wallace writes paper on band structure of graphene
- 2004 K.S. Novoselov et al. realize and identify graphene experimentally

scotch

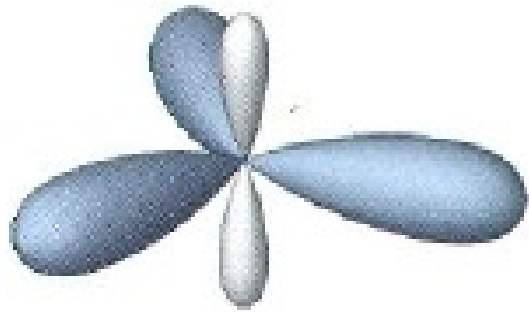


+

Imaging

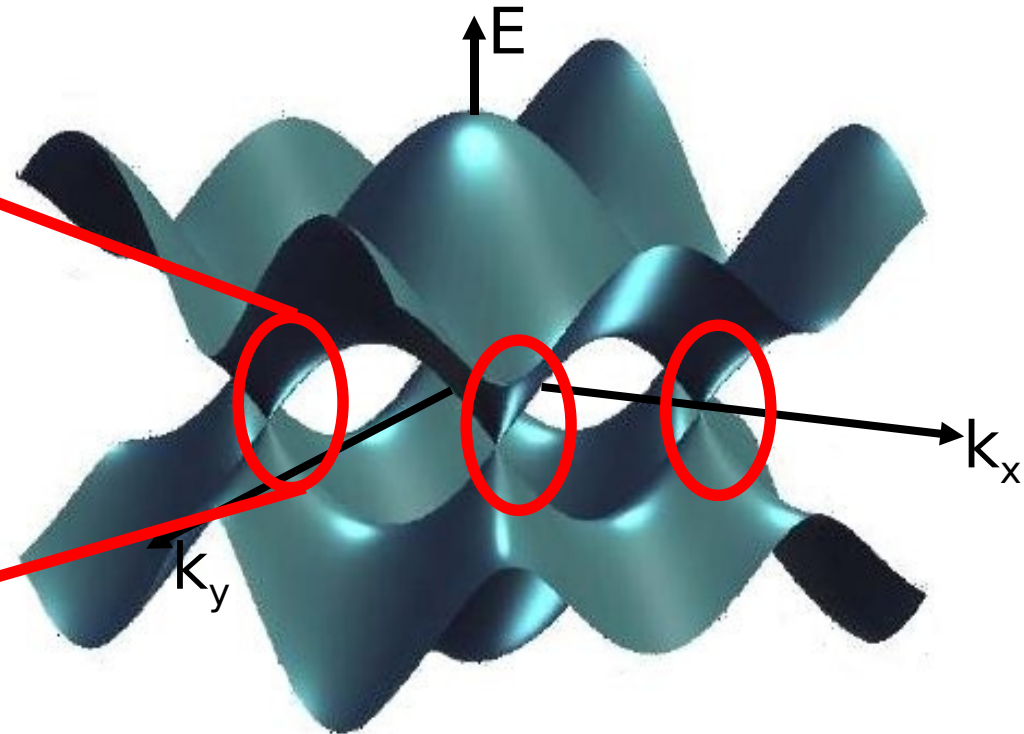
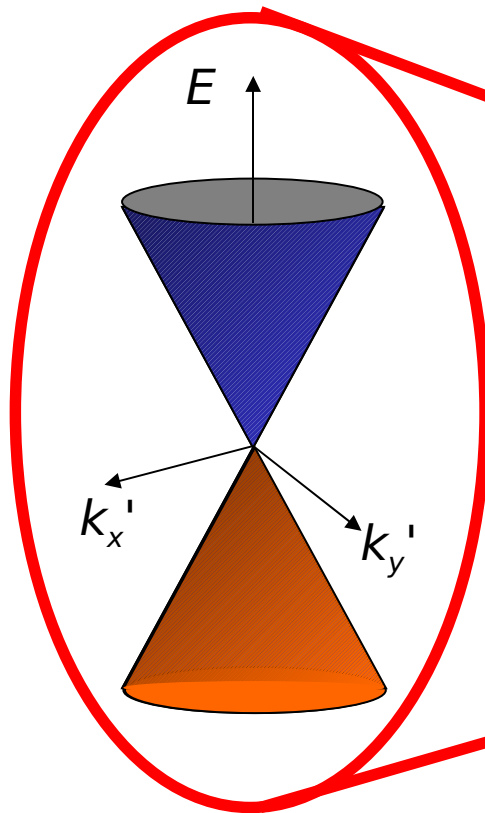


Band structure



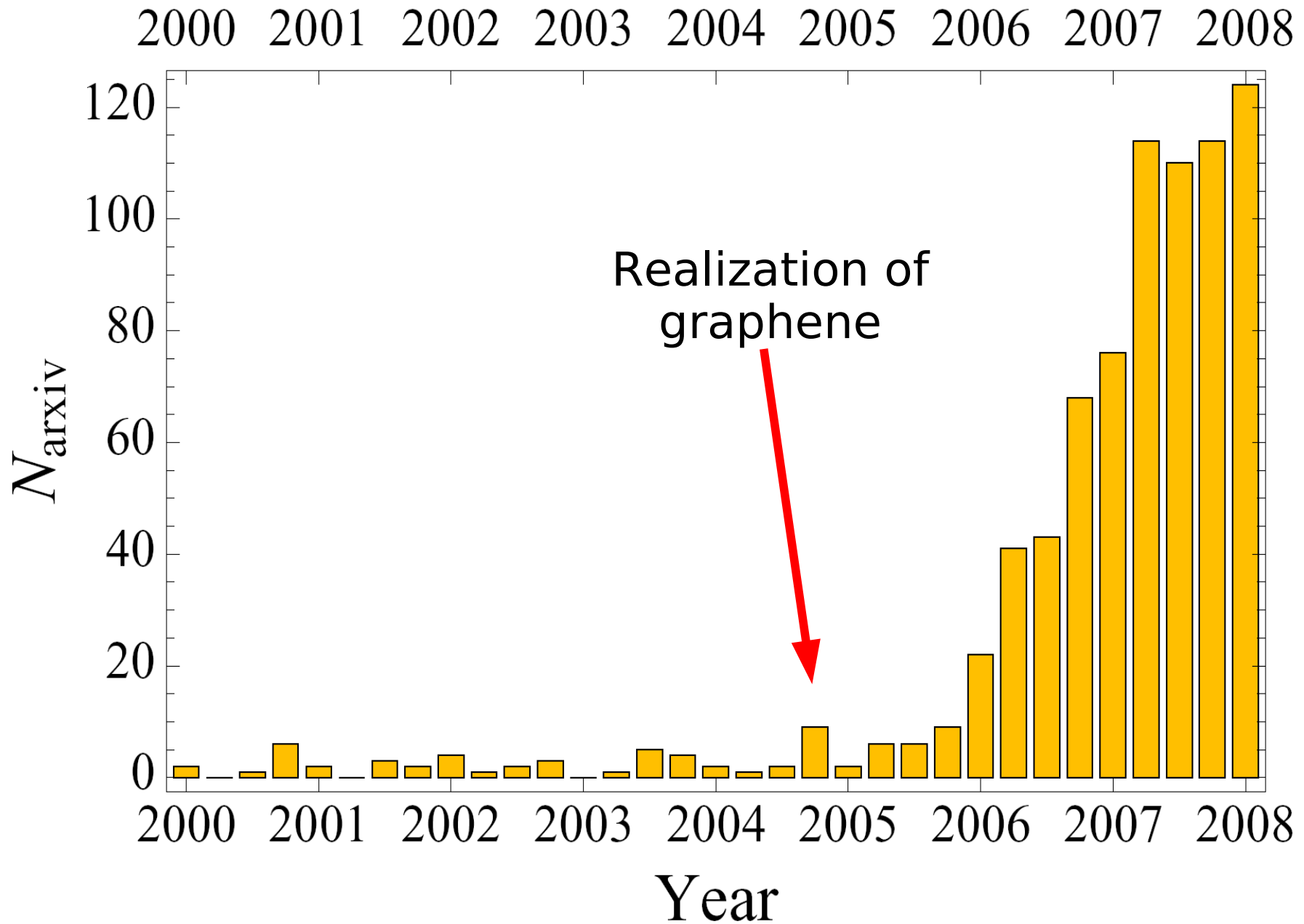
Carbon atom orbitals.

Tight binding model,
P. R. Wallace (1947)



Graphene has 2D Dirac cones

Is it interesting?



**Why is graphene so
interesting?**

2D Crystal

ABSENCE OF FERROMAGNETISM OR ANTIFERROMAGNETISM IN ONE- OR TWO-DIMENSIONAL ISOTROPIC HEISENBERG MODELS*

N. D. Mermin[†] and H. Wagner[‡]

Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York

(Received 17 October 1966)

Phys. Rev. Lett.
17, 1133(1966)

Crystalline Order in Two Dimensions*

N. D. Mermin[†]

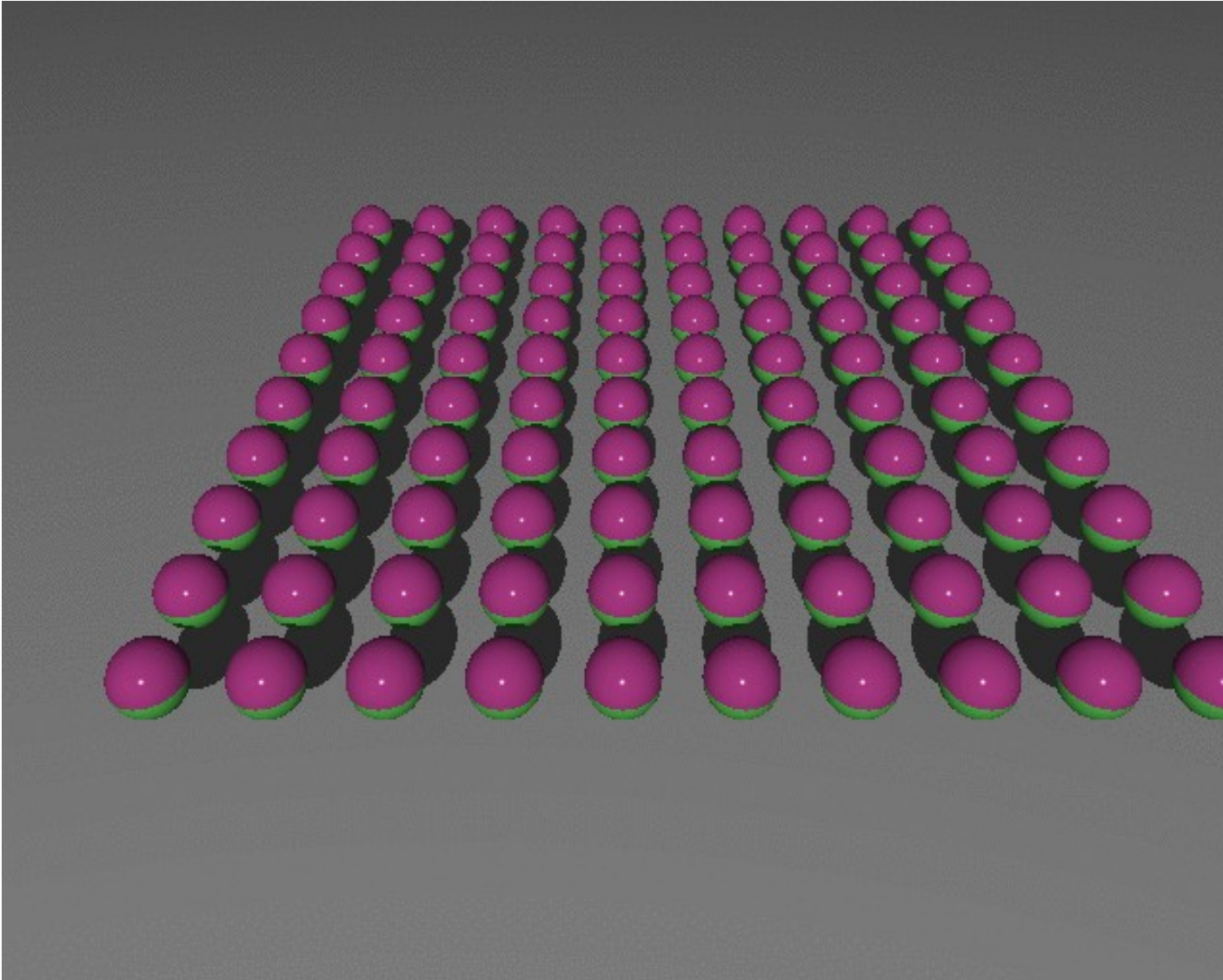
Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York

(Received 1 July 1968)

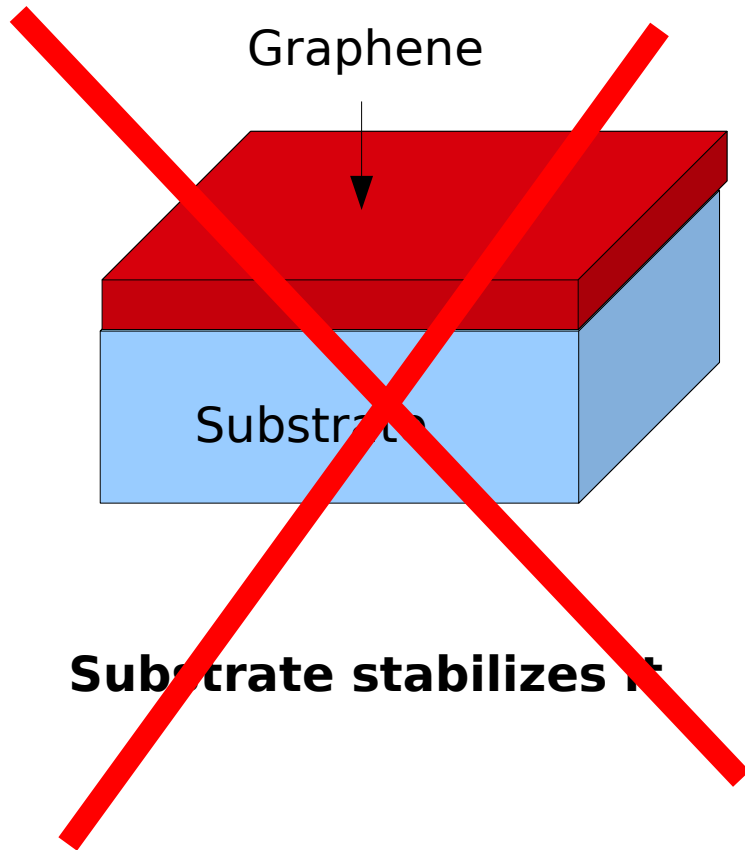
This result excludes conventional crystalline long-range order in two dimensions

Phys. Rev. **176**,
250 (1968)

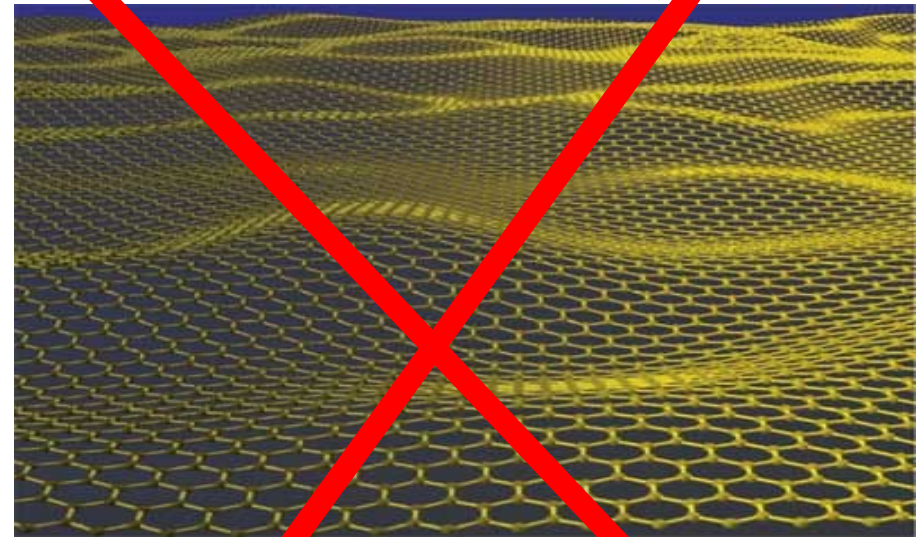
Fluctuations in 2D destroy the lattice



Why graphene can exist



Substrate stabilizes it



Ripples are present

Finite Size!

“...the bound can be so weak to allow two-dimensional systems of less than astronomic size to display crystalline order.”

Mermin Phys. Rev. (1966)

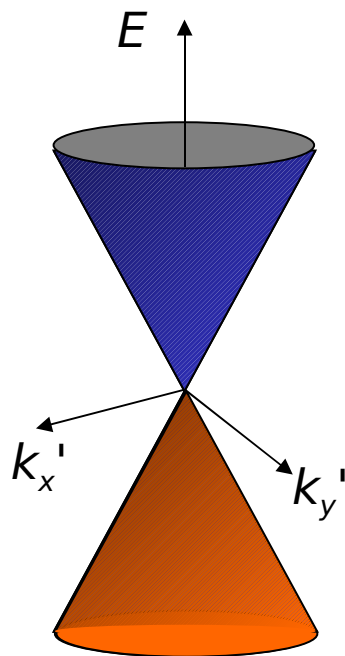
$$L < 10^{30} \text{ m.}$$

R.C. Thompson et al.
arxiv:0807.2938

Graphene is the ultimate flatland!

Dirac cones in graphene

From tight binding model we have that at the corners of the BZ the low energy Hamiltonian is:



$$H = \hbar v_F \sigma \cdot \mathbf{p}$$

Chiral Massless Dirac Fermions

Electrons obey laws of 2D QED!

The Fermi velocity v_F is $\sim 1/300$ the speed of light c . We have **slow** ultrarelativistic electrons.

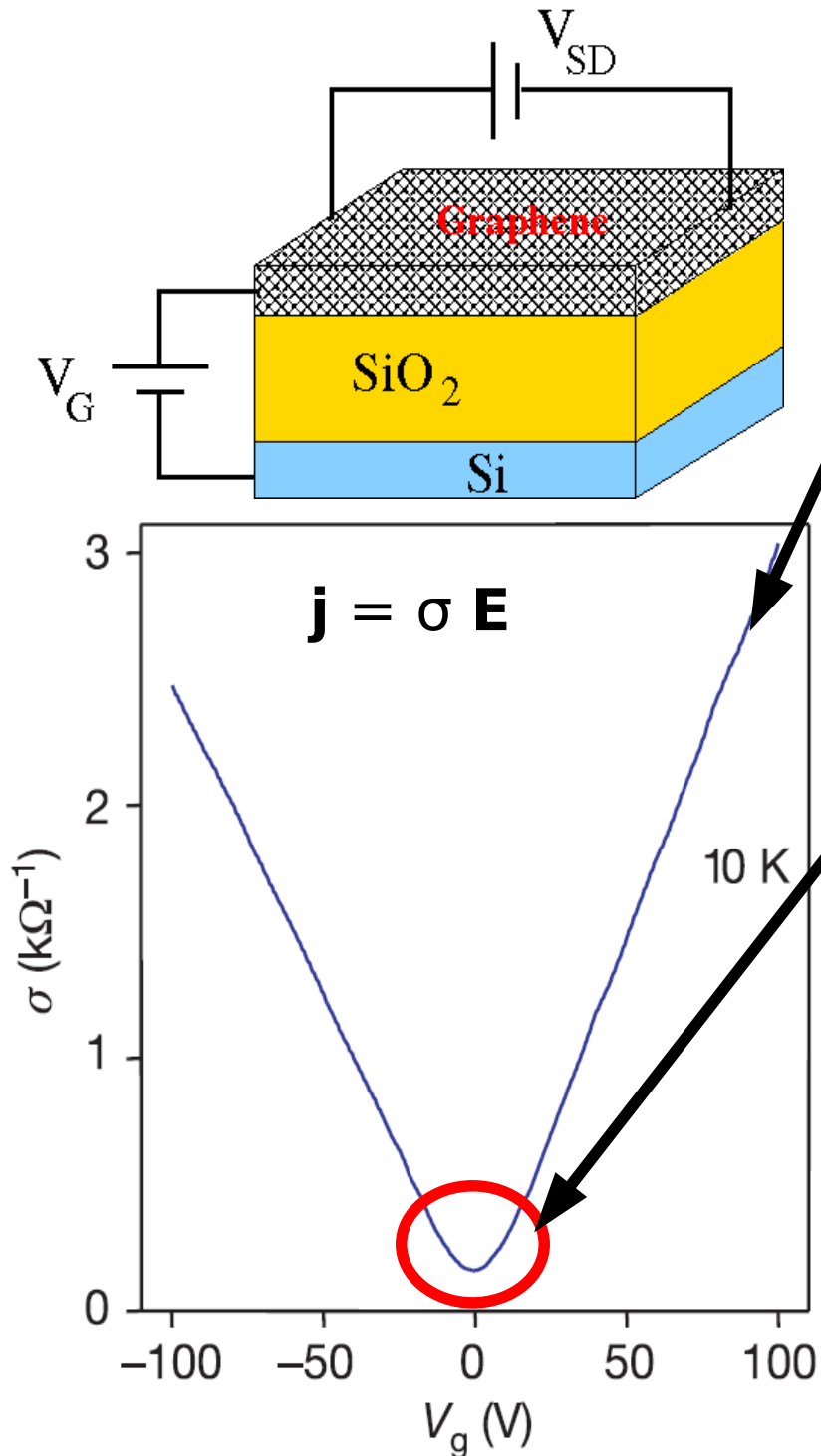
2D !

QED with a pencil and some scotch!

Experimental consequences

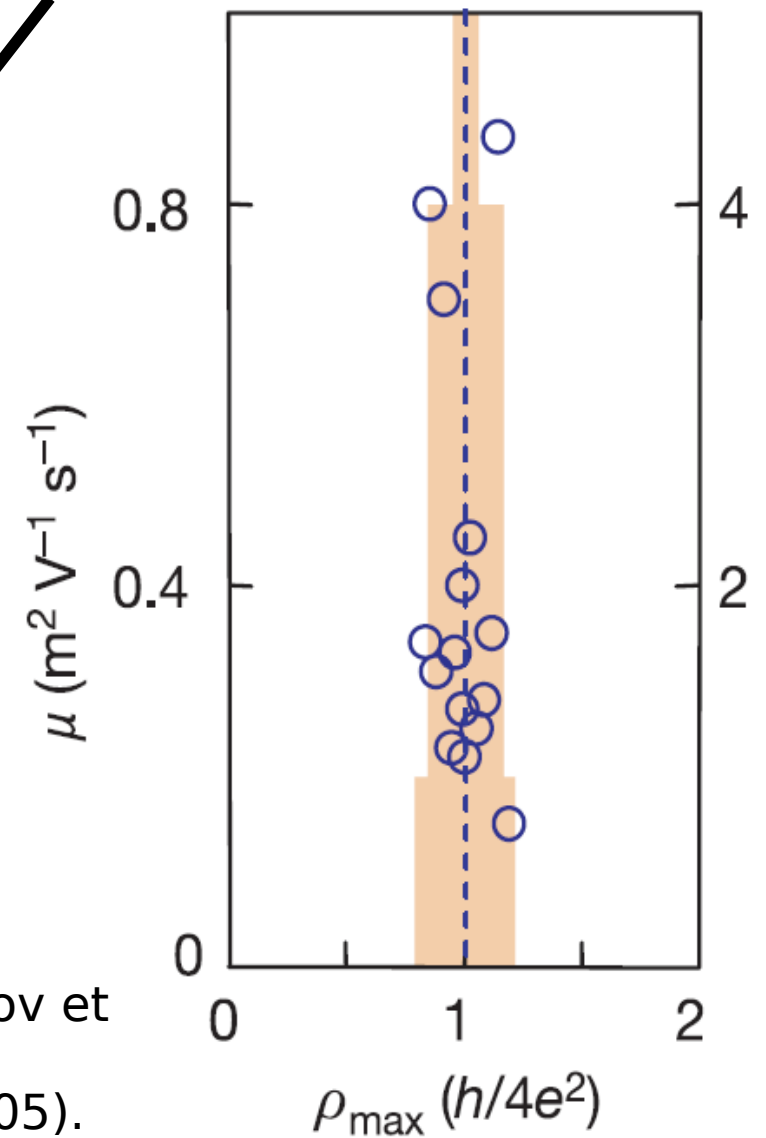
- Direct experimental observation of Dirac cones in ARPES experiments;
- Unusual half-integer Quantum Hall Effect;
- **Puzzling transport results.**

Electronic Transport: Questions



1) Linear scaling with doping: $\sigma \sim n$

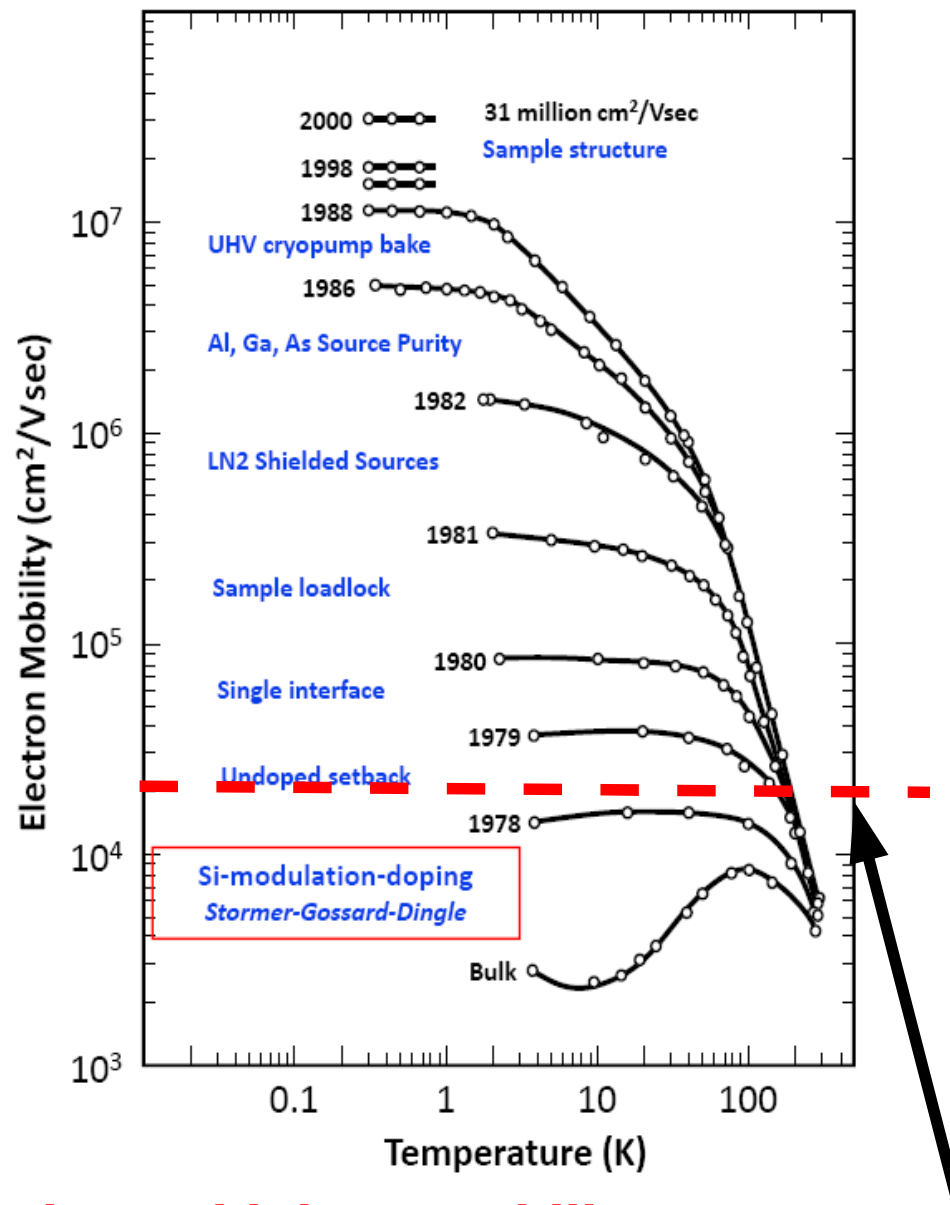
2) Minimum of conductivity for $n \rightarrow 0$.



K. S. Novoselov et al., Nature **438**, 197 (2005).

Toward a Graphene Pentium processor

Mobility: $\mu = \sigma / (e n)$



High mobility

Good!

Minimum of conductivity

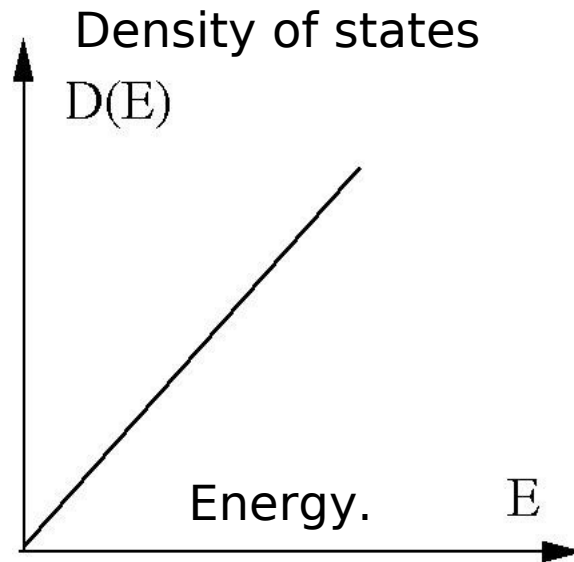
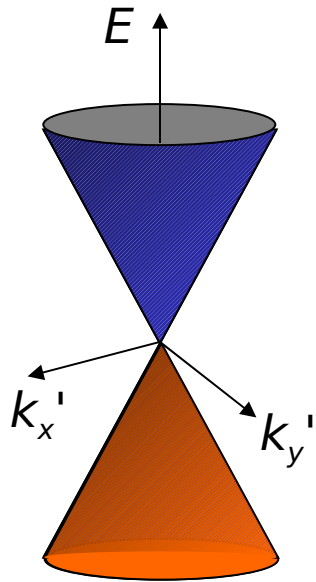
Bad!

K.S. Novoselov et al,
Science **306**, 666 (2004)

Y. Zhang et al.
Nature **438**, 201 (2005)

Graphene: highest mobility at Room T.

Why minimum of conductivity is puzzling



$$\sigma \propto D(E)\tau$$

$$\tau \propto \frac{1}{D(E)}$$

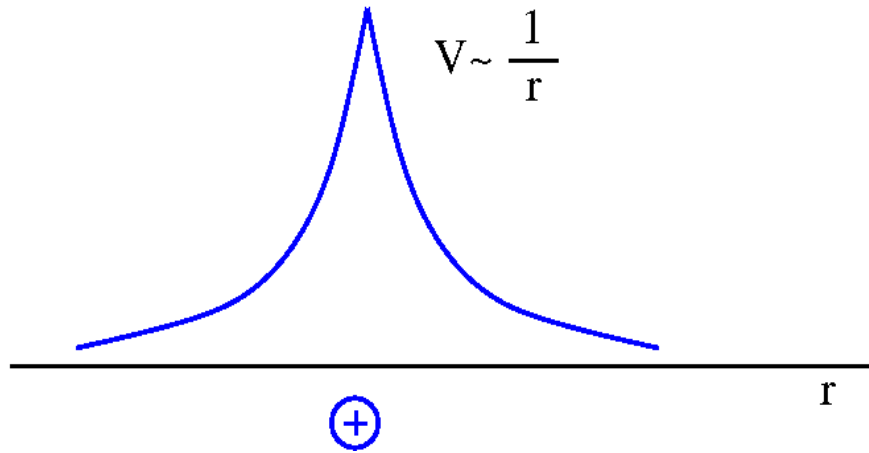
$$\sigma_{\min} = \begin{cases} 0 & \text{Intervalley scattering} \\ \infty & \text{Disorder that does not mix the valleys} \\ \frac{4e^2}{\pi h} & \text{No disorder. Different from experiment.} \end{cases}$$

Experimentally: $\sigma_{\min}$ is a sample dependent constant !

1) Linear relation between σ and n

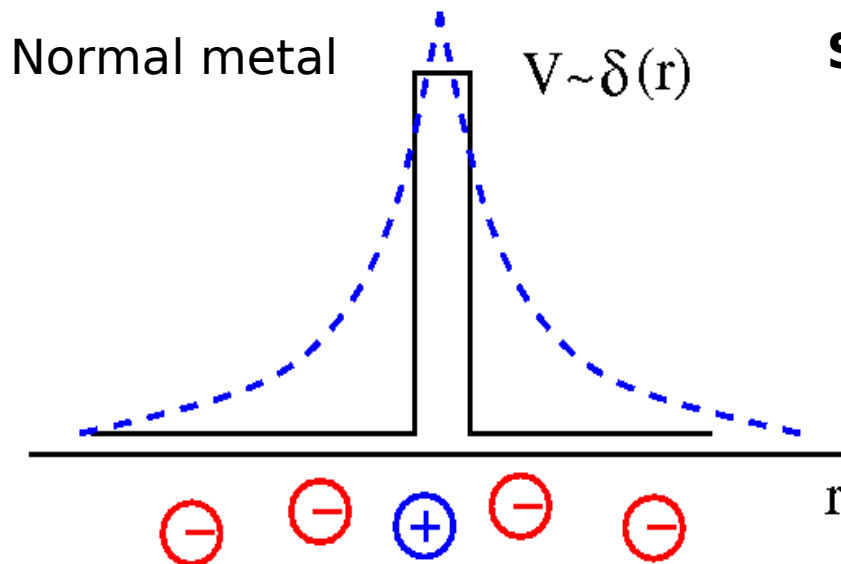
We need a scattering potential that gets stronger as n becomes smaller

Charge impurities

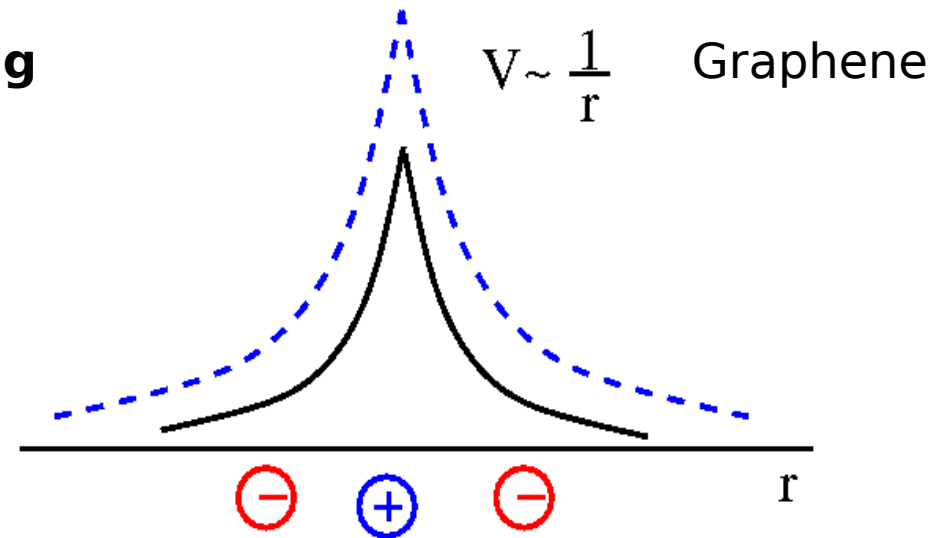


$$\sigma \propto \frac{|n|}{n_{\text{imp}}}$$

T. Ando J. Phys. Soc. Jpn (2006);
K Nomura & MacDonald PRL (2006);
E. Hwang et al. PRL (2007)



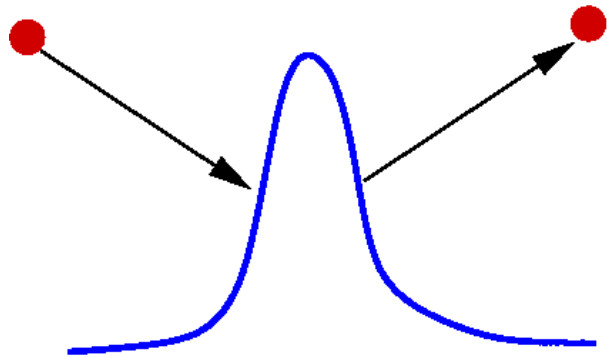
Screening



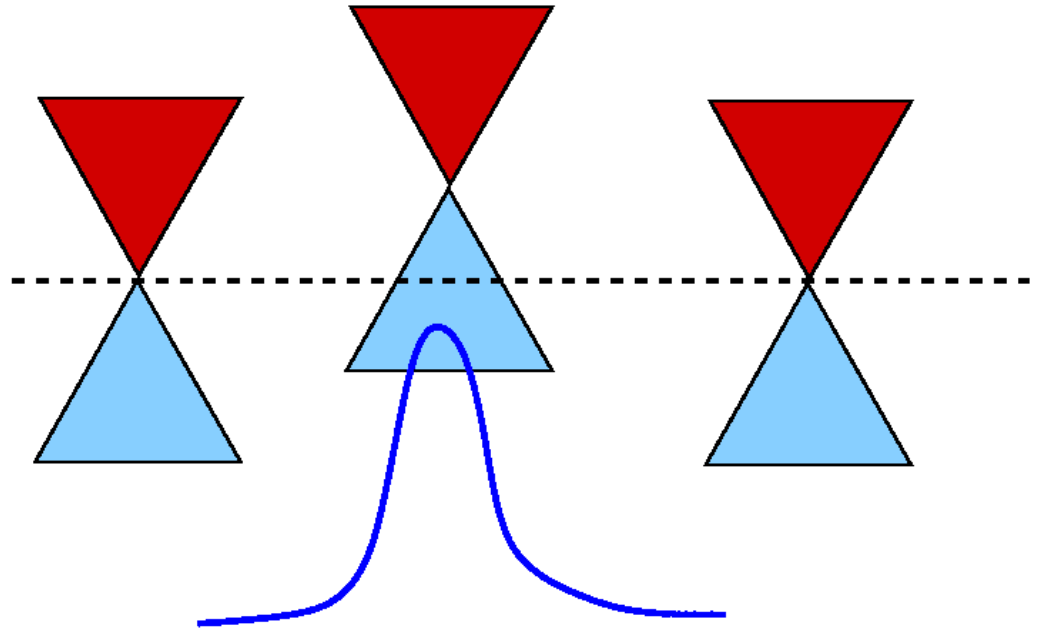
Close to the Dirac point graphene
is a poor screener

The finite minimum conductivity remains unexplained

Effect of disorder



Scattering



Shifts bottom of the band $\longleftrightarrow$ shift of Fermi energy $\delta n(r)$

$$n(r) = \langle n \rangle + \delta n(r)$$

At the Dirac point
 $\langle n \rangle = 0$



The density
fluctuations
 $\delta n(r)$
dominate the physics

Thomas-Fermi-Dirac theory

Developed a theory that is:

- **Microscopic**
- **Nonperturbative**

It includes:

- **Disorder potential due to charge impurities;**
- **Nonlinear screening;**
- **Exchange and correlation effects;**

Build the energy functional $E[n]$, $n(\mathbf{r})$ is the carrier density .

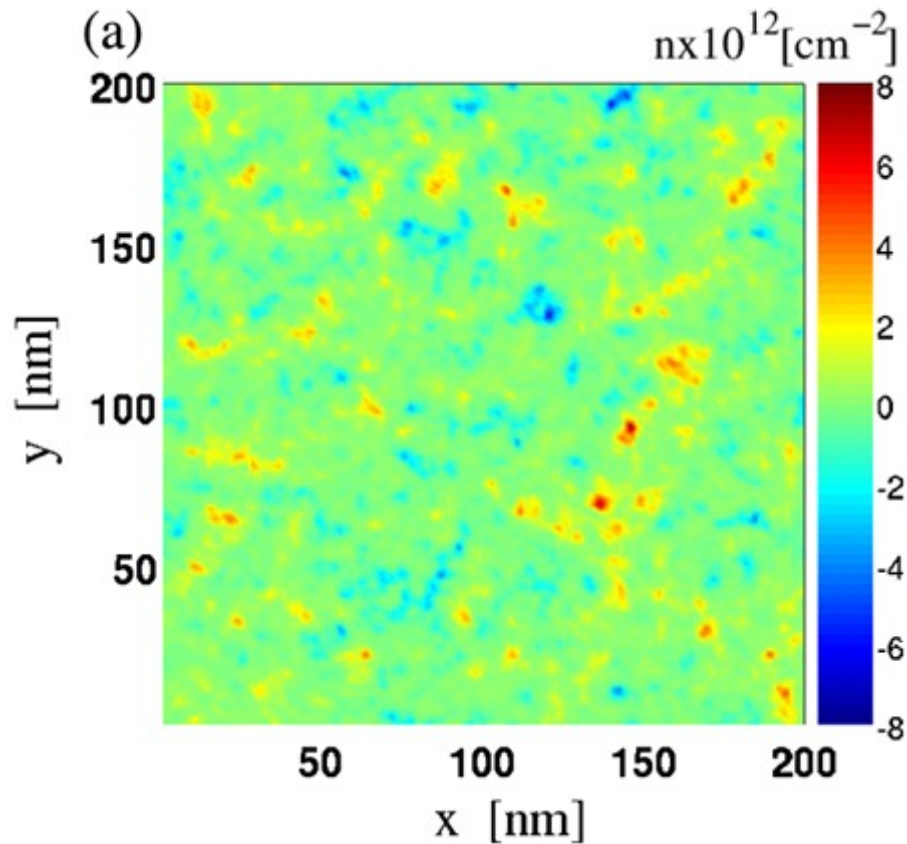
$$E[n] = E_{kin}[n(\mathbf{r})] + E_H[n(\mathbf{r})] + E_{exch}[n(\mathbf{r})] - \int_A \mathbf{V_D} n(\mathbf{r}) d^2r - \mu \int_A n(\mathbf{r}) d^2r$$

$\mathbf{V_D}$ is the disorder potential generated by charged. The only inputs are:

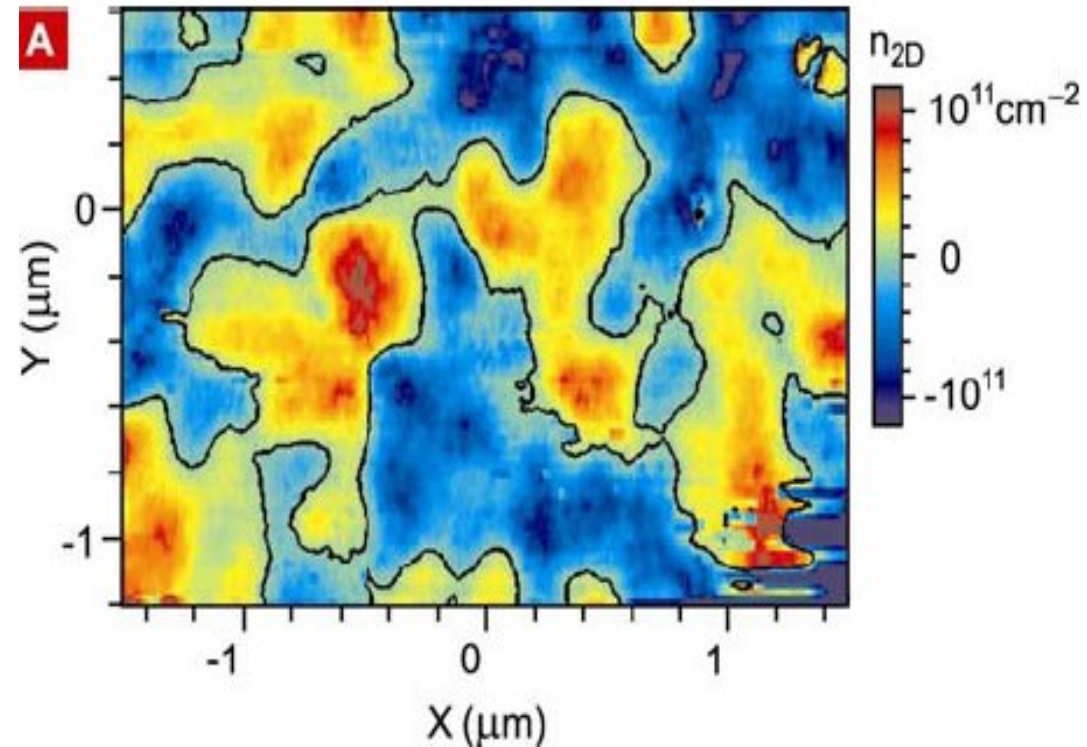
- The charge impurity density n_{imp}
- Their average distance, d , from the graphene layer

both reliably extracted from transport experiments at high doping.

Dirac point: single disorder realization



ER and S. Das Sarma,
Phys. Rev. Lett. **101**, 166803 (2008)



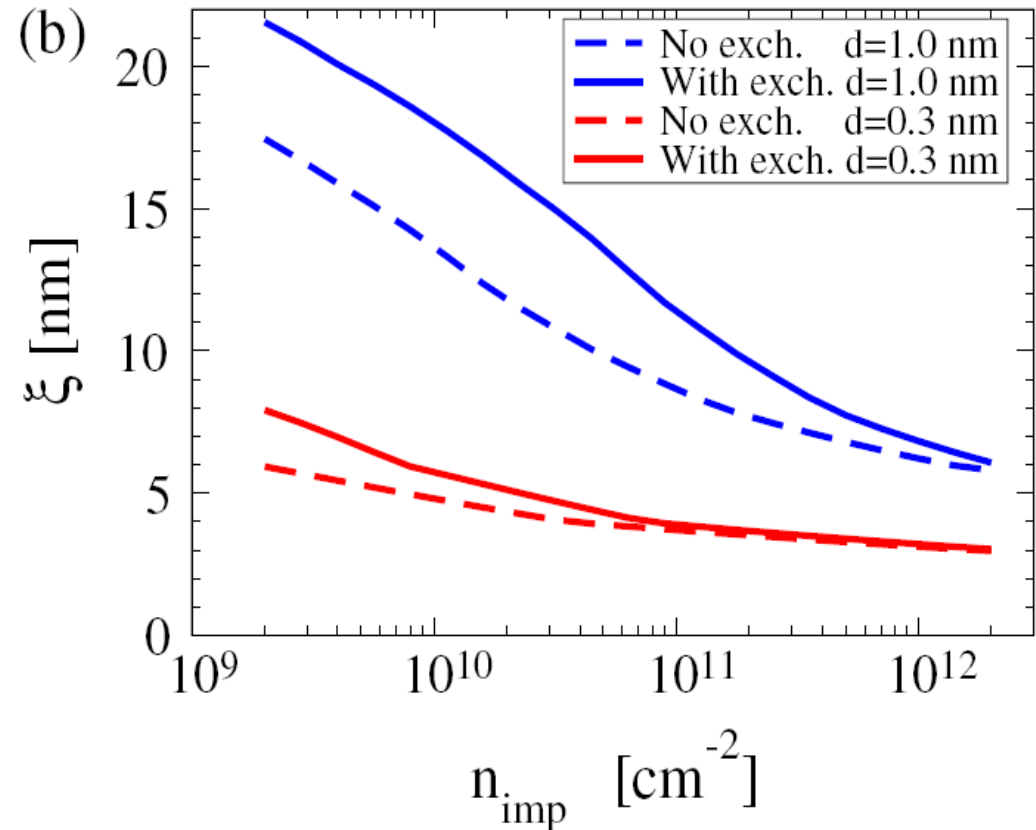
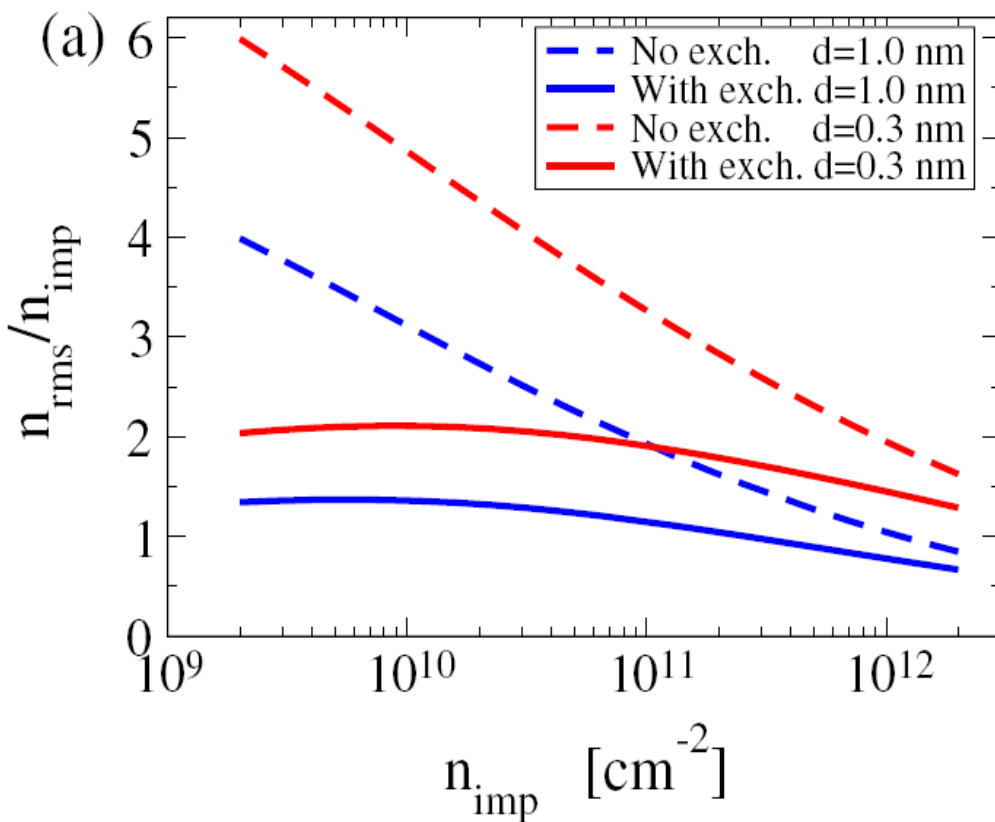
J. Martin et al.,
Nat. Phys. **4**, 144 (2008)

Disorder breaks the carrier
density landscape in electron-
hole puddles

Disorder averages results at the Dirac point

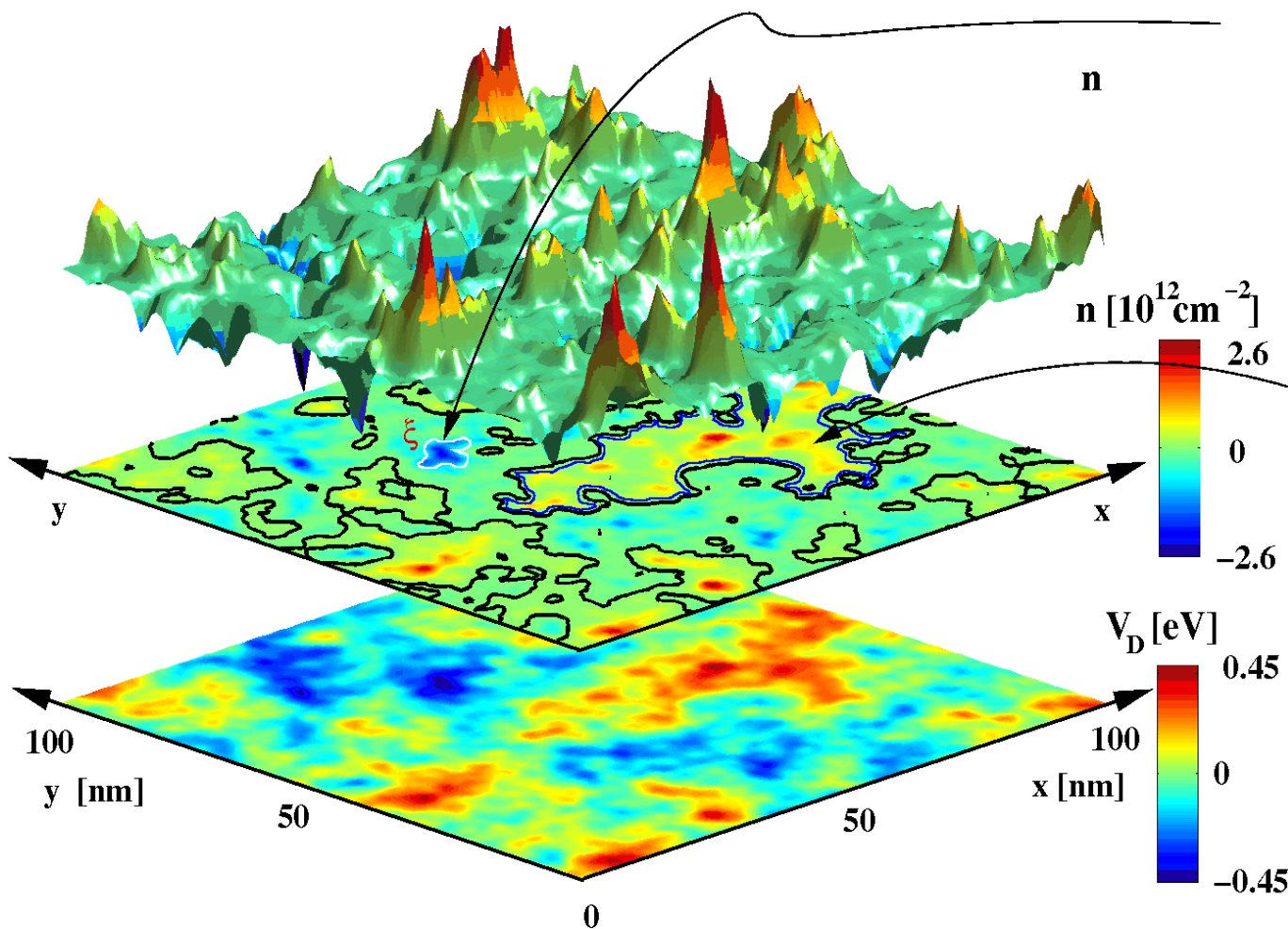
Density fluctuations
root mean square: n_{rms}

Correlation length: ξ



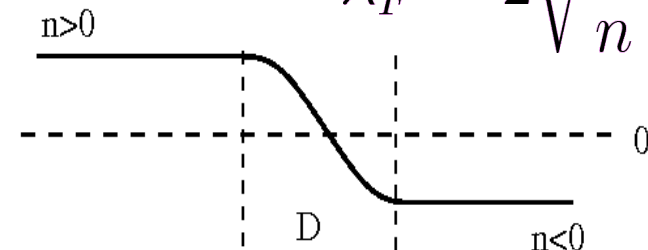
Carrier density properties

Small region of size ξ , ~ 10 nm, **fixed by non-linear screening**, and high density. $\delta Q \sim 2e$. Result in agreement with recent STM experiment [V. Brar et al. unpublished]



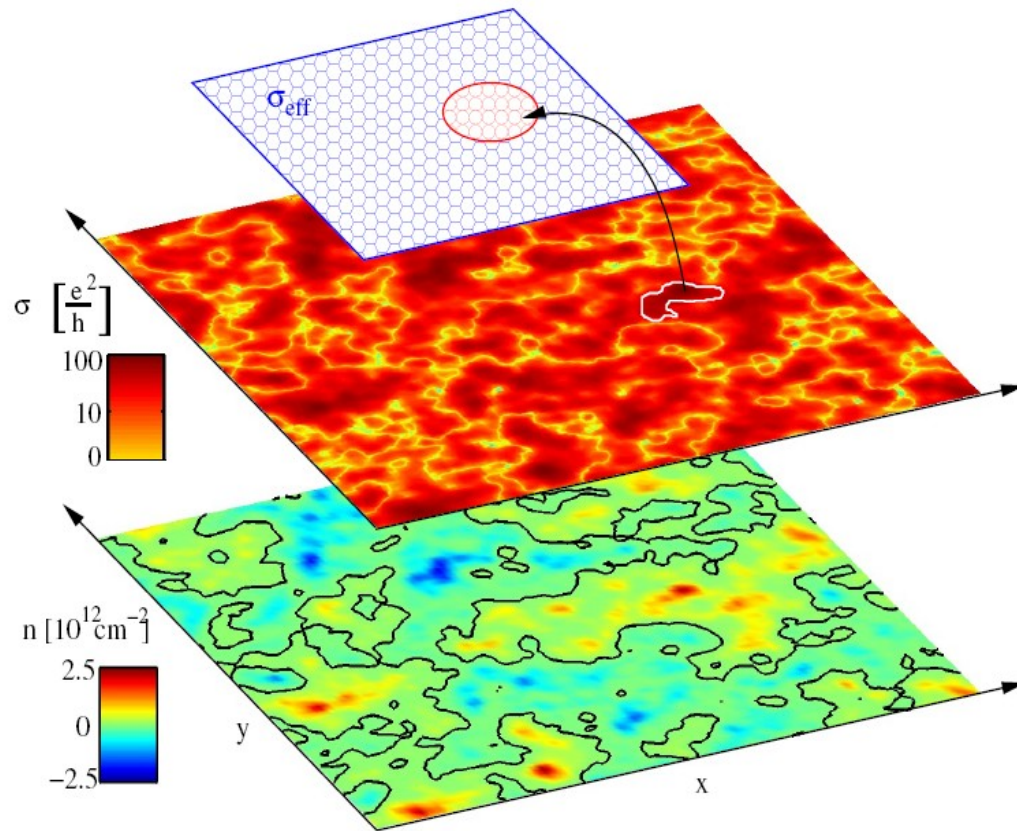
Wide regions of size $\sim L$ (sample size) and low density. $\delta Q \sim 10e$.

The density across the electron-hole puddles boundaries (p-n junctions) varies on length scales, D , of the order of

$$\lambda_F = 2\sqrt{\frac{\pi}{n}}$$


Inhomogeneous conductivity

The inhomogeneous character of the n will be reflected in inhomogeneous transport properties such as the conductivity, σ .



$$\sigma \propto \frac{|n|}{n_{\text{imp}}}$$

Locally

Natural approach:
Effective Medium Theory.

[Bruggeman Ann. Phys
(1935), Landauer J. Appl.
Phys. (1952)]

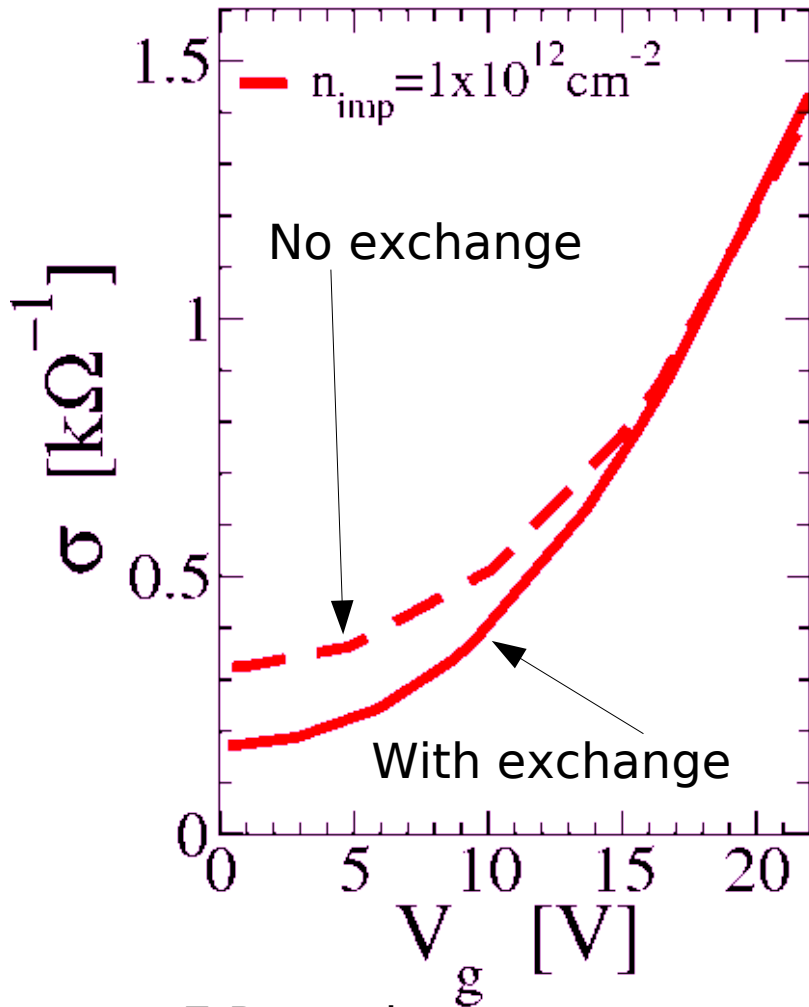
Can we use it in graphene?

Two conditions:

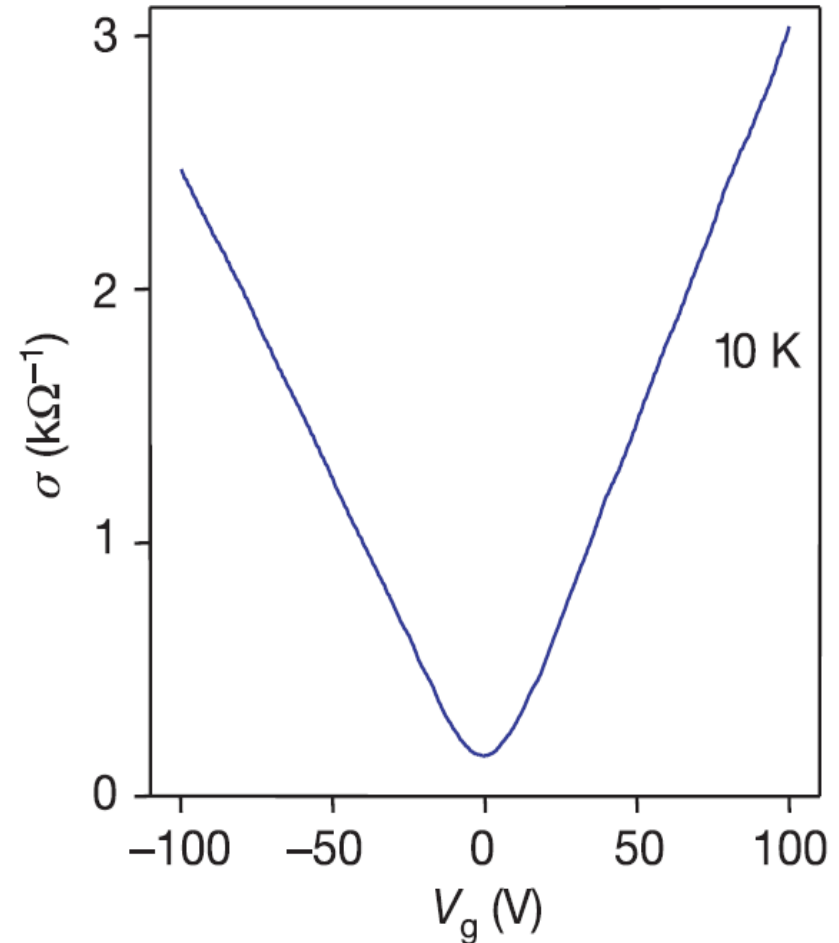
- 1) The mean free path, l , must be much smaller than the size of the homogeneous regions
- 2) The resistance across the homogeneous regions must be much smaller than the resistance inside the regions.



Conductivity vs. gate voltage



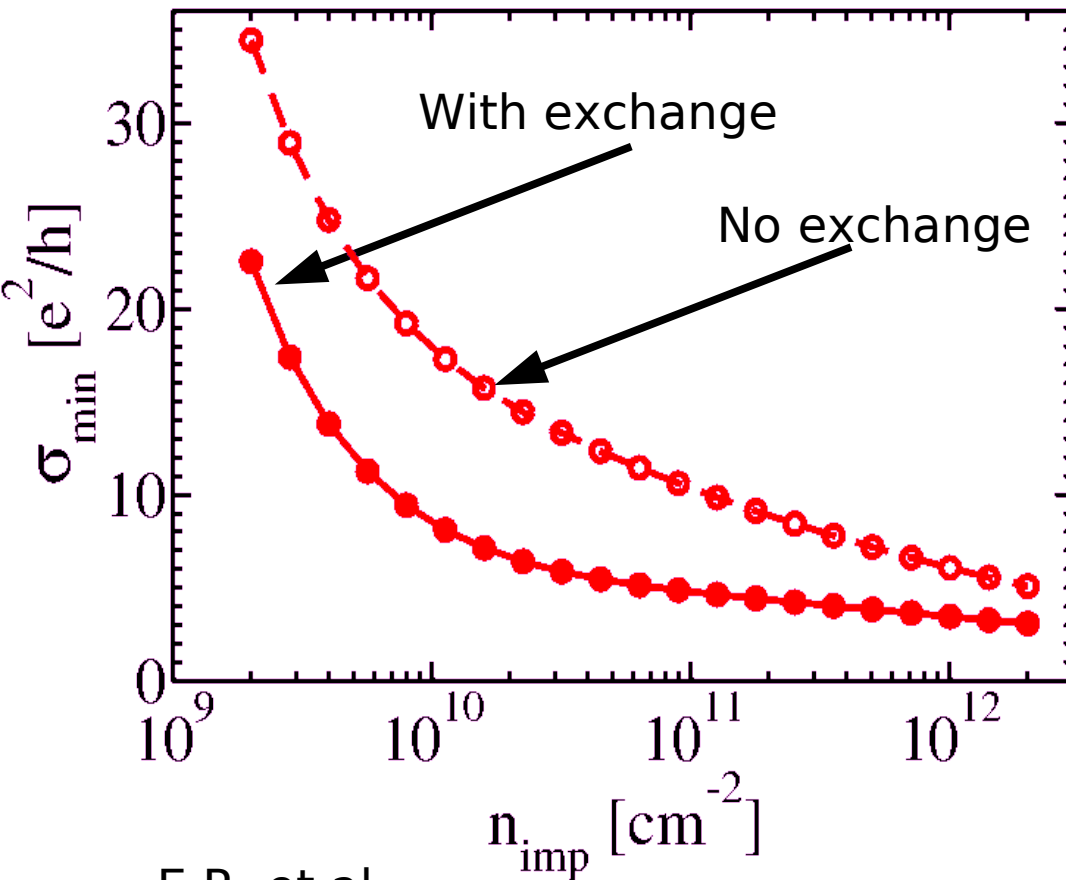
E.R. et al.,
arxiv.0809.1425 (2008)



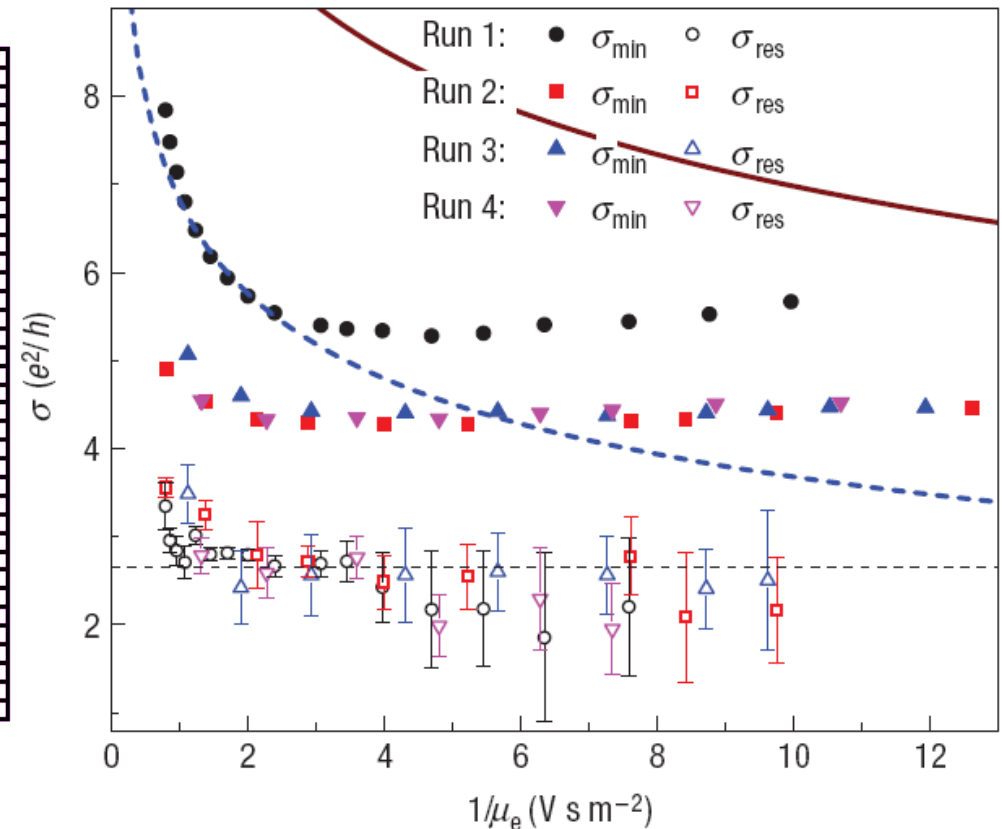
K. S. Novoselov et al.,
Nature **438**, 197 (2005).

- Finite value of the conductivity at Dirac point;
- Recovers linear behavior at high gate voltages;
- Describes crossover;
- Shows importance of exchange-correlation at low voltages.

Minimum conductivity vs. impurity density



E.R. et al.,
arxiv.0809.1425 (2008)

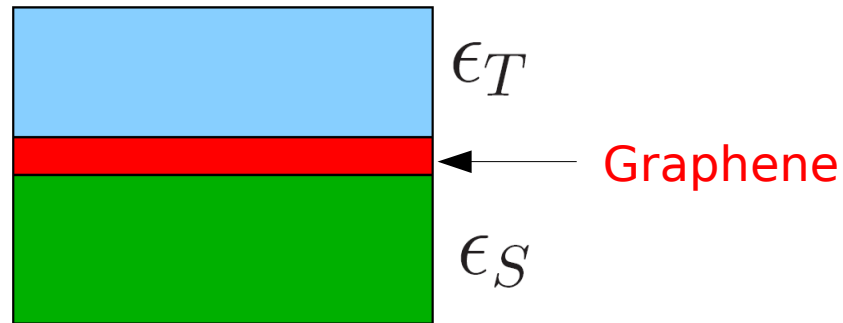


J.-H. Chen et al.
Nat. Phys. **4**, 377 (2008)

Dependence of conductivity on impurity density in qualitative and quantitative agreement with experiments.

Tuning the fine-structure constant, r_s

$$r_s = \frac{e^2}{\hbar v_F \bar{\epsilon}}$$



$$\bar{\epsilon} = \frac{\epsilon_T + \epsilon_S}{2}$$

$r_s : 0.5 \rightarrow 1$

C. Jang et al.
Phys. Rev. Lett.
101, 146805 (2008)

- r_s controls: **strength of disorder**; **strength of interaction**, **exchange**.



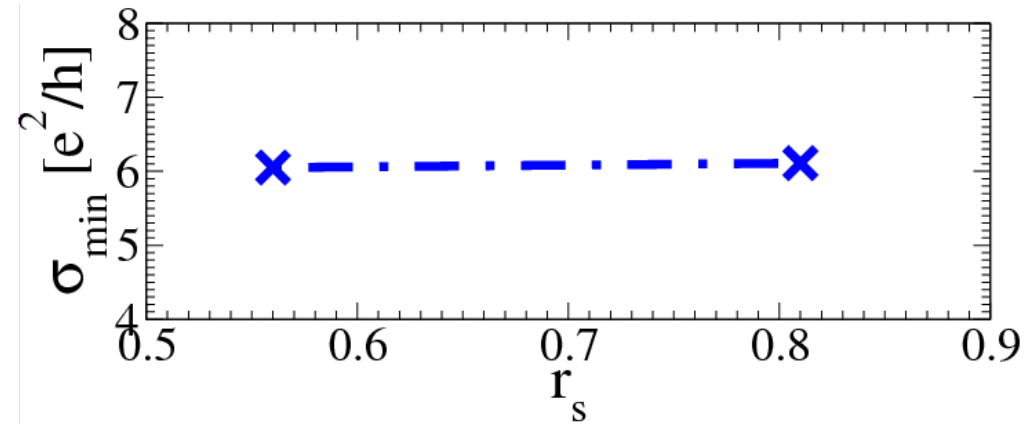
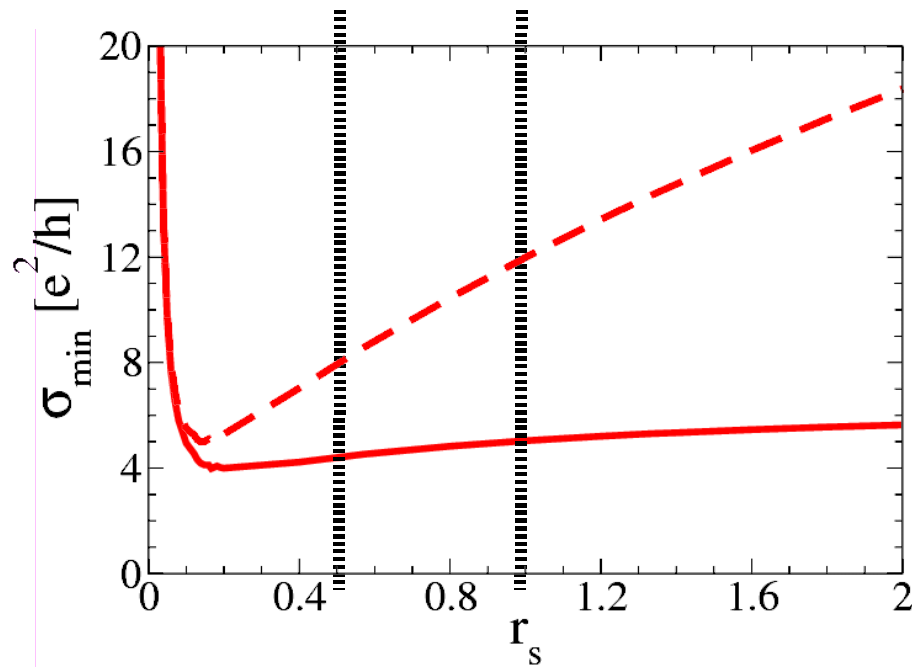
Strongly affects the density profile.

- r_s controls **strength of scattering**.

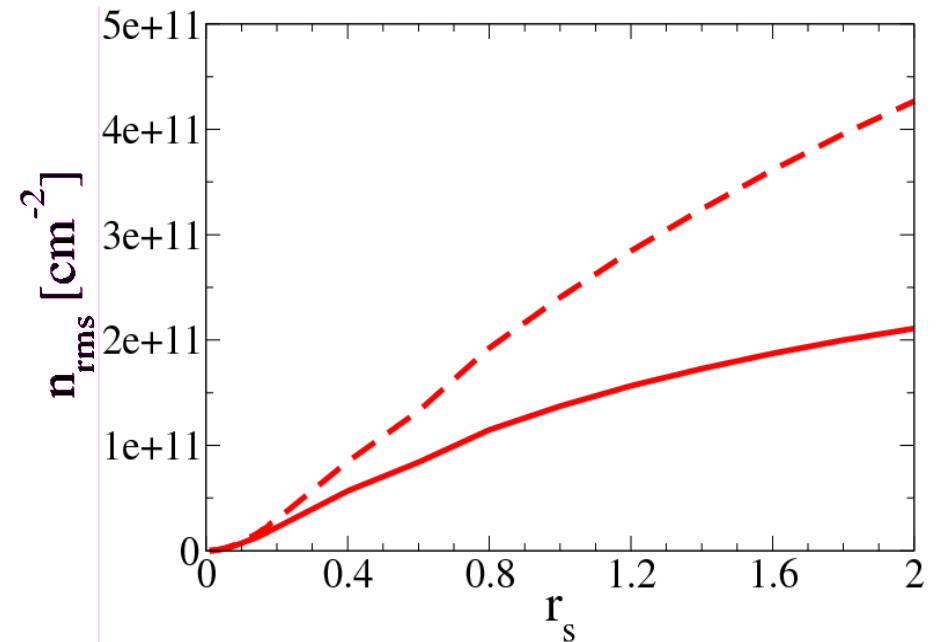
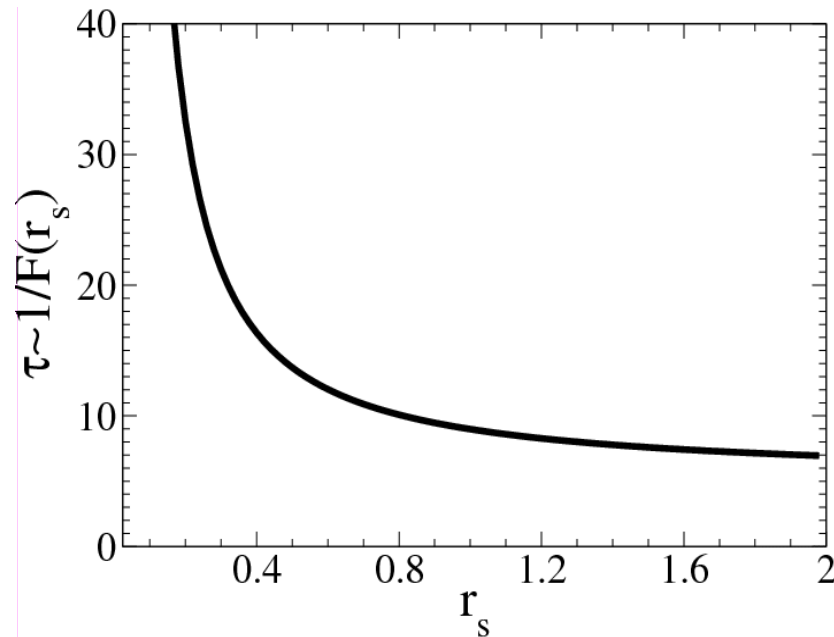


Affects scattering time

r_s dependence of the minimum conductivity



C. Jang et al.
Phys. Rev. Lett. **101**, 146805 (2008)



Conclusions

- Graphene has many interesting properties
- Unusual transport properties: minimum of conductivity
- Presented microscopic and non-perturbative theory to characterize strong density fluctuations
- Showed how strong density fluctuation explain the minimum of conductivity.

Graphene unique playground to learn about 2D massless Dirac electrons

Many interesting physical phenomena.

Great potential for technological applications.