A Study of Concert Acoustics: Understanding Phi Beta Kappa Hall

A thesis submitted in partial fulfillment of the requirement for the degree of **Bachelor of Science** in **Physics** from the College of William and Mary in Virginia,

by

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I. Abstract

Phi Beta Kappa (PBK) Hall has the reputation among musicians from the College of William & Mary (and beyond) of having inferior acoustics, being described as "very dry." The purpose of my senior research project was to 1) understand the acoustics of PBK Hall through both experimentation and theoretical modeling, and 2) to suggest possible ways to improve the acoustics of the Hall.

II. Introduction

In my early research to understand the principles of acoustics, I ran across the works of Leo Beranek, one of the early pioneers in the field of acoustics and writer of several books on the subject. After perusing his general Acoustics textbook, I focused in on my particular topic, concert hall acoustics, from his more recent book "Concert and Opera Hall: How They Sound" (Beranek 1996). In this book he introduces a list of measurable factors which allows one to make a judgment of a concert hall's acoustics.

One of the primary factors is *Reverberation Time* (RT), defined as *the time, multiplied by a factor of two, that it takes for the sound in a hall to decay from –5 to –35dB below its steady-state value.* The factor of two is necessary in order to make RT conform to the original definition of sound decay which was from 0 to -60dB (Beranek 1996, p568). RT was defined to be recorded in a fully occupied hall, measured in the few seconds of quiet periods following orchestral stop-chords recorded during live concerts, normally at one or two points in the audience. In Beranek's book, a listing of seventy-six concert halls was compiled, and of those halls considered "dry" or "dead," the average length of the reverberation time (RT) was around 1.3 seconds. Of those halls considered "live," the average RT is 2.0 sec. Reverberation Time, however, is difficult to test easily because it requires a fully occupied theatre (and thus a large ensemble performance with an audience that fills PBK). *Early Decay Time* (EDT) is somewhat easier to measure, because it is normally recorded in an unoccupied hall. EDT is defined as *the length of time, multiplied by a factor of six, that it takes for a sound to decay 10dB after the instant the sound source is turned off*. The factor of six is necessary to make the EDT time comparable to RT. EDT should not be confused with *Early Sound*, which describes the direct sound and those reflections that take place within 80ms after the arrival of the direct sound (Beranek 1996, p28). For unoccupied halls, EDT is normally an average of sound measurements taken at a number of different frequencies, such as 125, 250, 500, 1,000, 2,000, and 4,000 Hz, from eight of more different locations around the hall, with one to three locations of the sound source on stage. The EDT for a concert hall with upholstered seats (such as PBK) is about 0.2s longer than the RT (Beranek 1996, p54).

The *strength factor* G_{mid} (dB) is a measure of sound intensity at seats in a hall from a loudspeaker source that has a known power output, averaged over two frequencies, 500 and 1000 Hz, and at 8 to 20 positions in the hall.

The *initial-time-delay gap* t_I is the time it takes for the first non-direct sound to reach the listener. The preferred value for t_I is around 20ms when measured in the center of the main floor.

The *bass ratio* BR is the ratio of two reverberation times for an occupied hall. The denominator is the average of the RT's at 500 and 1,000Hz and the numerator is the average of the RTs at 125 and 250Hz.

David Dudley, the Technical Director of PBK Hall, remarked that the Hall was designed with Theatre productions in mind, and therefore efforts were made to indeed the reverberation time, given that in a theatrical setting less reverberation time is preferred in order to keep speech understandable. However, he also agreed that the acoustics of PBK were less than ideal, supporting the idea that there are other factors than reverberation time that determine a positive listening experience. Thus, although I measured the reverberation time, and it was lower than the average "acoustically pleasing" concert hall, I decided to focus on other factors that determine a hall's acoustic appeal.

I also wished to use blueprints of PBK Hall and, if certain acoustical reflections were noticeable, to speculate about the source(s) of such strong (and acoustically undesirable) reflections.

Acoustical tests lend themselves well to Fourier analysis, because rooms typically behave differently for high and low frequencies (pitches), yet all complex sounds are a mixture of many frequencies. Fourier's Theorem essentially states that any periodic wave can be written as a superposition of waves with multiples of a fundamental frequency. Specifically,

$$f(t) = \sum_{n=1}^{\infty} A_n \sin(n\omega t + \phi_n) \qquad \textit{(Equation 1)}$$

where ω is the fundamental angular frequency of the wave, A_n is the amplitude of the nth sine wave, and Φ_n, is the phase of the *nth* sine wave. Typically this series is truncated, however, as n$\rightarrow\infty$, *Equation 1* is exact. Fourier analysis allows the separation of a sound wave into its constituent frequencies, the fundamental and its overtones. The Fast Fourier Transform (FFT) is a computationally fast method of doing a discrete Fourier transform, and is often used by audio analysis software. The FFT method samples the wave form over a time $\Delta t >> \omega$ and extracts the frequency components present in that interval. The larger Δt, the larger the range of frequencies accurately deduced. However, if the waveform is changing within the time Δt, the greater frequency distinction comes at the cost of less accuracy of frequency intensity with respect to time. The number of samples used in this project was either 256 (256/48,000Hz = ~5ms accuracy) or 2048 (2048/48,000Hz = ~43ms accuracy).

III. Descriptions of Method

A. Types of Experiments:

This thesis describes three types of experiments conducted in PBK Hall from Sept 2002 through April 2003:

1) The playing and recording of a sound impulse in order to observe the arrival times of strongly reflected sounds.

2) The playing of a pure sine wave from the stage, and observing its intensity from various points in the audience. This checks for standing wave interference.

3) The playing of a sustained sound that is abruptly cut off in order to ascertain the EDT.

B. Experimental sessions:

Three data-recording sessions were conducted at PBK Hall, at which various aspects of the three main experiment types were carried out. For a listing of the experiments conducted by date, see Table 1.

Date	Type of Experiment	Description of Experiment	Purpose of Experiment
10/25/02	Impulse	Multiple dropping of a 8.5"x13" spiral-bound notebook from shoulder height at Point A, as recorded from Points 1-6 (see Figure 1)	To test out recording equipment / procedures, to gather first data sets and analyze reflected sound peaks to correlate with predicted reflective surfaces.
10/28/02	Impulse	Multiple hitting of a 12"x12" piece of plywood against the floor by hand, as recorded from Points 1-6	To improve on previous impulse experiment by creating louder, more focused impulse, yielding better sound peaks
	Interference	Playing a static single-source sine wave from stage and recording its intensity from various parts in the Hall	To test for varying levels of intensity where it should be the same, which would reveal interference patterns caused by standing waves
	EDT	Saturating the room with a sine wave, then cutting it off and recording decay from Points 1-6	To measure the EDT and also to test Fast Fourier Transform method
3/21/03	Impulse	Playing a CD on a boombox on stage containing a 1) 1000Hz 1ms, 2) 1000Hz 5ms, 3) noise 1ms, and 4) noise 5ms impulse	To improve on previous impulse experiment by creating a shorter, more controlled impulse
	EDT	Playing a CD on a boombox on stage containing computer-generated sine waves of 1) 125Hz, 2) 250Hz, 3) 500Hz, 4) 1000Hz, 5) 2000Hz, 6) 4000Hz, 7) noise	To improve on previous EDT experiment with a louder sound source, a more controlled sine wave, and to confirm data gathered by FFT method.

Table 1 *– Listing and brief description of experiments conducted at PBK sorted by date of experiments. “*Impulse” *refers to an experiment in which there is a short spike of sound intensity followed by silence, “*Interference*” refers to an experiment to measure levels of constructive or deconstructive interference, and “*EDT*” refers to an experiment measuring Early Decay Time.*

C. Summary of Impulse Experiments:

The purpose of these sets of experiments was to create a large sound impulse on stage and then record the time taken by subsequent sound peaks following the initial impulse, which would allow me to calculate the distance traveled by the sound through non-direct paths and match it with the distances estimated by reflections off of certain key objects in PBK such as walls or ceilings, thus showing sources of unwanted reflections.

I began with my first experiment (*Experiment 10/25/02*), whose primary purpose was to test my data collection methods and equipments. I decided to record an impulse (short length) of sound projected from a source on stage to a recording device in six different points around the hall (see Figure 1). From the impulse data I was also going to analyze the impulse, the subsequent decay, and attempt to record how long it took for subsequent strong reflections, and

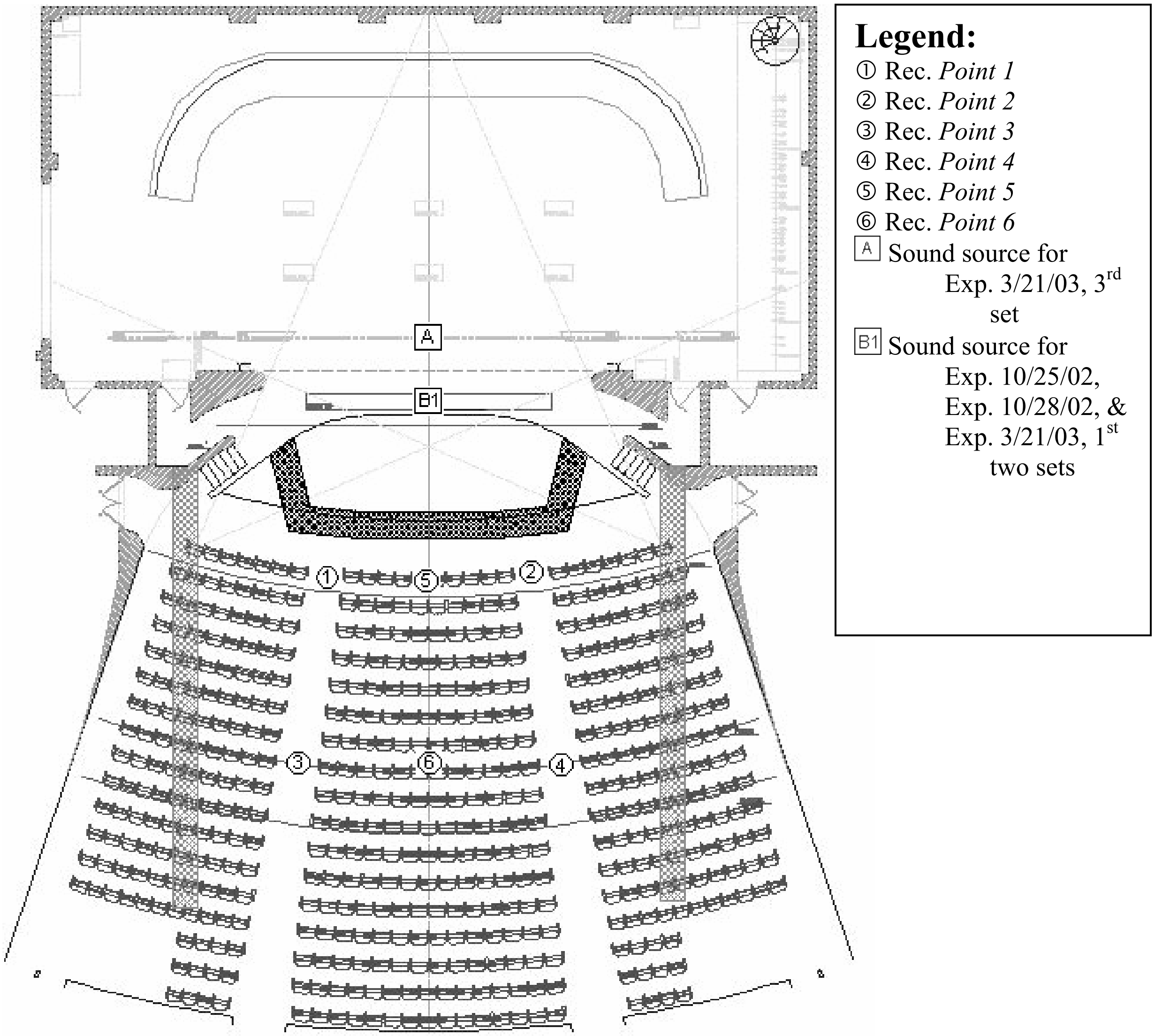


Figure 1 *– Top-down (or "Bird's Eye") view of PBK Hall*

therefore extrapolate distances from solid reflecting surfaces (for more details of each experiment, see the **Data Collection and Analysis** section).

From that experiment I realized that the source used, an 8.5"x13" spiral-bound notebook dropped from shoulder height, was an inaccurate, errant, and insufficiently powerful source for an impulse. I later returned to PBK (*Experiment 10/28/03)* with a 12"x12" plywood board as the sound source to hopefully provide a louder and more focused sound, but found that while the new impulse was stronger, the sound was still not as focused as it could have been. For the final experiment at PBK Hall (*Experiment 3/21/03*) I created a CD that contained (among other things) four sets of computer-generated sound impulses, of the properties 1) 1ms @ 1000Hz, 2) 5ms @ 1000Hz, 3) 1ms of random noise, and 4) 5ms of random noise, and played it on a higher-intensity speaker system than used previously.

D. Summary of Interference Experiment:

I performed one experiment to test for constructive/destructive interference, which for static sound waves is caused by parallel surfaces such as walls, floors, and ceilings interfering with each other and canceling out in certain fixed areas and reinforcing each other in certain other areas. In one part of *Experiment 10/28/03,* I played a sine wave from the stage and recorded the sound intensity (in dB) at six points (Points 1-6, see Figure 1), to look for variations in intensity in areas that should be the same (example – points symmetrical to each other) that would signal undesirable acoustical interference.

E. Summary of EDT Experiments:

The purpose of these sets of experiments was to determine the Early Decay Time (EDT) of PBK Hall, which is one acoustical property of a concert hall that is strongly related to its judgment as a good hall. To record the EDT, a sound source on stage was sustained until the intensity level was constant, and then the sound was cut off and the decay following the cutoff was measured. To get an accurate measurement of the sound intensity, a Fast Fourier Transform algorithm was used.

In the first experiment of this type (*Experiment 10/28/02)* I used a sine wave generator and a single speaker at Point B1, and recorded twice from Point 6. From this experiment I realized that a louder sound source had to be used, as well as a more tightly controlled sound source (cutting off the sine wave involved pulling the plug out of the signal generator, leading to a clicking sound that may have slightly skewed the sound cut-off point. So for the second and final experiment of this kind (*Experiment 3/21/03)* I created a CD that had computer-generated pure sine waves of the following frequencies: 1) 125Hz, 2) 250Hz, 3) 500Hz, 4) 1000Hz, 5) 2000Hz, and 6) 4000Hz. The CD was played on a boombox with a higher sound-intensity level then before, and led to more usable data.

Fourier analysis was used to measure the EDT (using a WAV editor that had a FFT function built into it), because of its ability to accurately measure the sound intensity of certain fixed frequencies (the sine waves used in my experiments). To test the Fourier analysis, after recording the pure sine waves, a computer-generated noise (representing a sound wave composed of many different frequencies) was also played and cut off in order to test and confirm the Fourier analysis for the noise would be similar to the Fourier analysis for the pure sine waves (as it should be theoretically).

F. Use of PBK Blueprints to Compare Experiments with Theory.

Data from *Experiment 03/21/03* and its analysis revealed that there were serious acoustical problems as indicated by a non-steady rate of decay. Therefore I used a blueprint of PBK Hall to roughly predict when certain strong (non-direct-source) reflections would occur, and tried to explain the uneven decay rate, which would seem to be much more of the cause of "unpleasantness" in PBK than previously thought. The method by which I derived approximations of the predicted reflections is as follows: opening up the PBK AutoCAD diagram with the AutoCAD viewer, I picked my sound source point, and then my recording point. Then, using both the side ("vertical") and the bird's eye ("horizontal") view, I identified potential paths by which sound could travel from the source point to the recording point. The viewer program would allow me to track distances in feet, which I then converted to meters. The first distance to be recorded would be the most direct path. Then I would find various sources that could bounce sound back. Once estimated, in order to find the time taken to travel the distance, I divided the measured distance by the speed of sound (330m/s at STP). Finally, in order to estimate the time of sound arrival after the initial pulse, I took that number and subtracted it from the time taken by the most direct path (See Table 2 in the **Data Collection and Analysis** section, under *Experiment 10/25/02* as an example).

IV. Data Collection and Analysis

A. Recording Equipment:

As my primary recording device I obtained a Canon ZR50 MC MiniDV digital camcorder, with an external Canon DM-50 Directional Stereo boom Microphone set not to just record from the shotgun, but also stereo from the sides ("Setting 1" on the microphone). The video/audio is recorded onto a MiniDV micro-cassette tape, for later digital input via Firewire port directly into my computer with zero loss of quality. I then extracted the audio from the AV file, which was set to record at a rate of 48,000 samples per second (Hz) and with a resolution of 16-bits per sample, thus allowing for a high quality (CD-quality is 44.1KHz, 16-bit) data sample.

For the experiment involving recording sound levels at various points in the Hall with a fixed source intensity (see *Experiment 10/28/03B*) to check for interference, I used a YFE YF-20 Sound Level Meter that recorded the sound intensity.

B. Analyzation Software:

For all of my Fourier analysis (Blackman-Harris FFT method with a sample size, depending on the experiment, of 256 or 2048), I used the professional WAV editor software

Sound Forge (Version 4.5c). Both the graphs of raw amplitude and the FFT data were then transferred to the Microsoft Excel 2002. For the graphical images of PBK Hall and also for measuring various distances in the Hall, I used a shareware AutoCAD viewer program called Vdraft CADview (Trial Version v2.21), which opened up the 2 PBK blueprints (in AutoCAD .DWG file format) given to me by the Theatre Department.

In the following sections I shall go through each experiment conducted in more detail and analyze the results.

C. Impulse Experiments:

To get initial data as a starting point and to test my data collection techniques, I decided to record an impulse (short length) of sound, projected from a source on stage, and to repeat the impulse six times from different recording positions in the hall (see Figure 1, each *Point* number corresponding to each position recorded). The source for *Experiment 10/25/02* was a notebook dropped from arm level to the floor (see Figure 1). I therefore set my recording device to record from the various points in the audience and then proceeded to drop my notebook in the same spot at least two times per recording position.

After getting the data onto my computer for WAV analysis, I realized that the initial impulse was not compact enough, perhaps caused by not all sides of the notebook hitting the stage at the same time, thus resulting in a more spread-out impulse, making a consistent measure of sound reflections quite difficult (See Figure 2 for examples of the recorded signal).

However, in my analysis I went ahead and attempted to pick out the loudest peak from the original impulse, and then also the points after the initial impulse where there was a noticeable peak in the amplitude (see Table 2).

Clip # / 6 (source location)	**1**	**1**	**2**	**2**	4	4	**5**	**5**	6	6	6
Take # w /in each clip	1	2	1	2	1	2	1	2	1	2	3
Time of Impulse Peak (s)	0.914	8.899	2.270	8.170	1.646	6.303	1.245	5.700	0.876	4.944	10.344
1st Impulse Peak in clip(s)	0.936	8.922	2.298	8.197	1.661	6.317	1.274	5.729	0.928	4.962	10.394
2nd Impulse Peak in clip(s)	0.963	8.948	2.323	8.226	1.697	6.339	1.302	5.753	0.990	4.977	10.498
3rd Impulse Peak in clip(s)	1.010	8.993	2.367	8.267	1.791	6.357	1.326	5.780	1.017	5.099	10.542
4th Impulse Peak in clip(s)	1.040	9.022	2.398	8.296	1.802	6.376	1.370	5.817	1.056	5.122	
5th Impulse Peak in clip(s)	1.062	9.046	2.420	8.322		6.448	1.393	5.842	1.077	5.144	
1st Impulse Peak (s)	**0.022**	**0.023**	**0.028**	**0.027**	0.015	0.014	**0.029**	**0.029**	0.052	0.018	0.050
2nd Impulse Peak (s)	**0.049**	**0.049**	**0.053**	**0.056**	0.051	0.036	**0.057**	**0.053**	0.114	0.033	0.154
3rd Impulse Peak (s)	**0.096**	**0.094**	**0.097**	**0.097**	0.145	0.054	**0.081**	**0.080**	0.141	0.155	0.198
4th Impulse Peak (s)	**0.126**	**0.123**	**0.128**	**0.126**	0.156	0.073	**0.125**	**0.117**	0.180	0.178	
5th Impulse Peak (s)	**0.148**	**0.147**	**0.150**	**0.152**		0.145	**0.148**	**0.142**	0.201	0.200	

Table 2 *– Estimation of sound reflections after an initial sound impulse from Experiment 10/25/03, analyzed from intensity vs. time graphs such as in Figure 2. 1st row is the clip number (corresponding to the Point number/location in Figure 1), 2nd row the specific take within that clip (up to three takes). The 3rd row indicates the time within each clip that the initial impulse peaks, and the subsequent five rows measure the times where there are noticeable peaks following the initial impulse. The last five rows contain the time from the first impulse to the specific peak, which is found by subtracting the initial impulse peak time from the specific peak given in the previous five rows. The clips in bold indicate data that should be nearly the same, Clips 1 & 2 (therefore Points 1 & 2) being symmetric to each other, and Clip 5 (Point 5) being in the same row geographically as Points 1 & 2 (see Figure 1); and indeed the numbers are similar.*

Then to do a proof of concept once I got the PBK Blueprints (in AutoCAD format, viewed using my AutoCAD viewer program), I decided to go ahead and estimate certain reflection points for the recordings done from the front row (*Points 1*, *2*, and *5* in Figure 1).

Again, while the estimates were inaccurate (due to the imprecise nature of the initial impulse), one can see a rough correspondence between the predicted reflection from the source point (SP) to the 2nd Hatching Block to the recording point (RP), from the SP to the lower back wall to the RP, and from the SP to the balcony bottom lip to the RP (see Figure 3 for the vertical diagram of PBK Hall with a labeling of the various surfaces and points, see Table 3 for estimated arrival times of reflected sound). As one can see, there is an error of ~5ms, which comes out to be a distance error of a little over 5 ft. (330m/s * 0.005s = 1.65m * 1ft / 0.3048m = 5.4ft). This error is quite large, but from rough estimation of the predicted reflection delay, this is an acceptable error (5ft/51ft = ~10% error, 5ft/77ft = ~6% error, 5ft/113ft = ~4% error). Something else to be taken into account as well, I physically measured some dimensions from PBK Hall and compared them with the blueprints I was given, and found some error (a 30ft measurement was off by 1ft) associated with the blueprints. Although the tools for an accurate predictive model were not available to me for this research project, in latter experiments I began to attempt to limit the error in the recorded measurements.

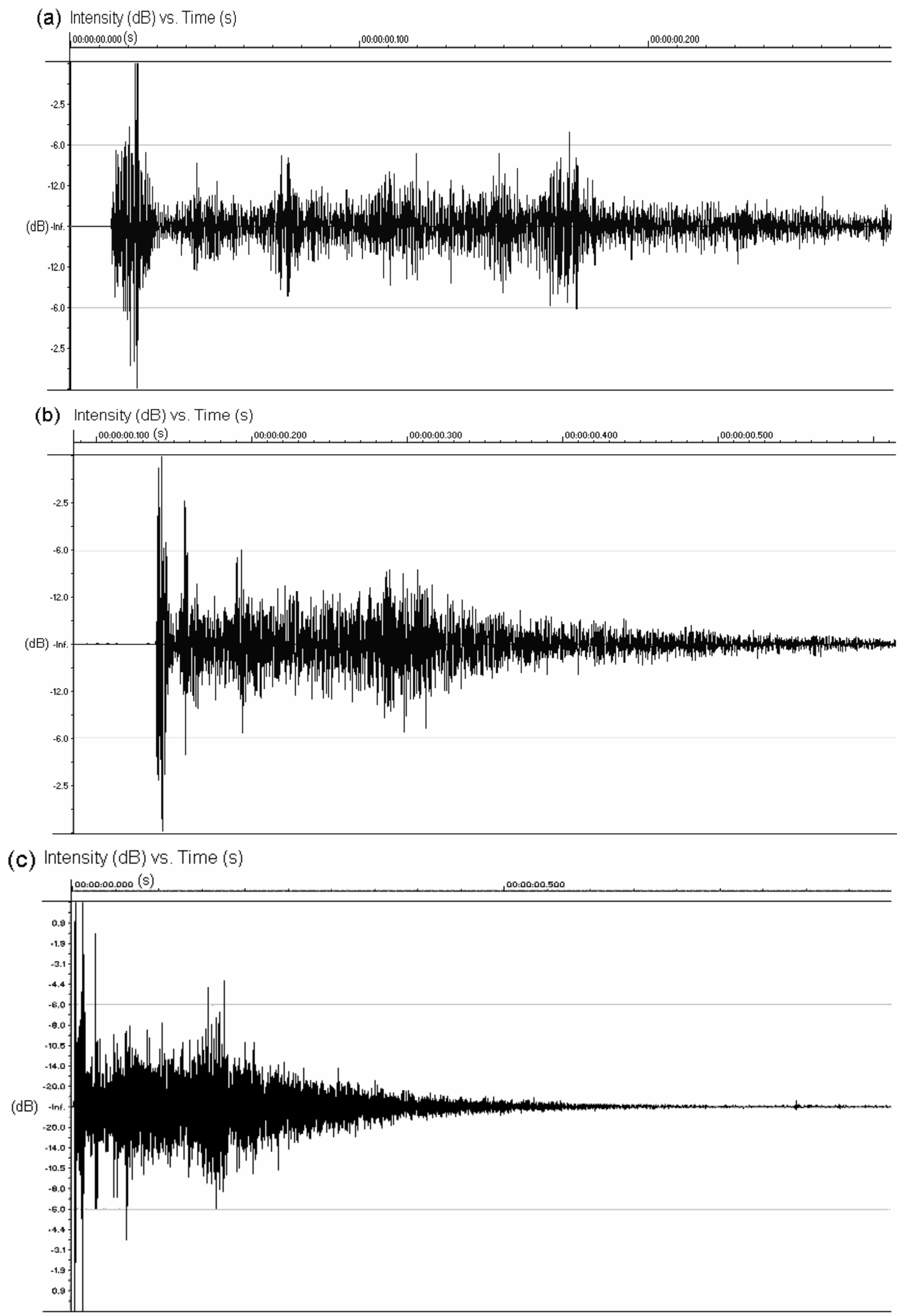


Figure 2: *Sound intensity vs. time recordings from the* Experiment 10/25/02 *impulse tests*, *showing an initial sound impulse followed by subsequent sound peaks. These peaks correspond to large reflections of the initial impulse by a uniform surface. The sources of all three recordings are the same – an 8.5"X 12" spiral-bound notebook dropped from shoulder height at point B1 (see Figure 1). They are recorded from (a) Point 2, (b) Point 4, and (c) Point 1 (see Figure 1). Note the long time span of the initial impulse in (a).*

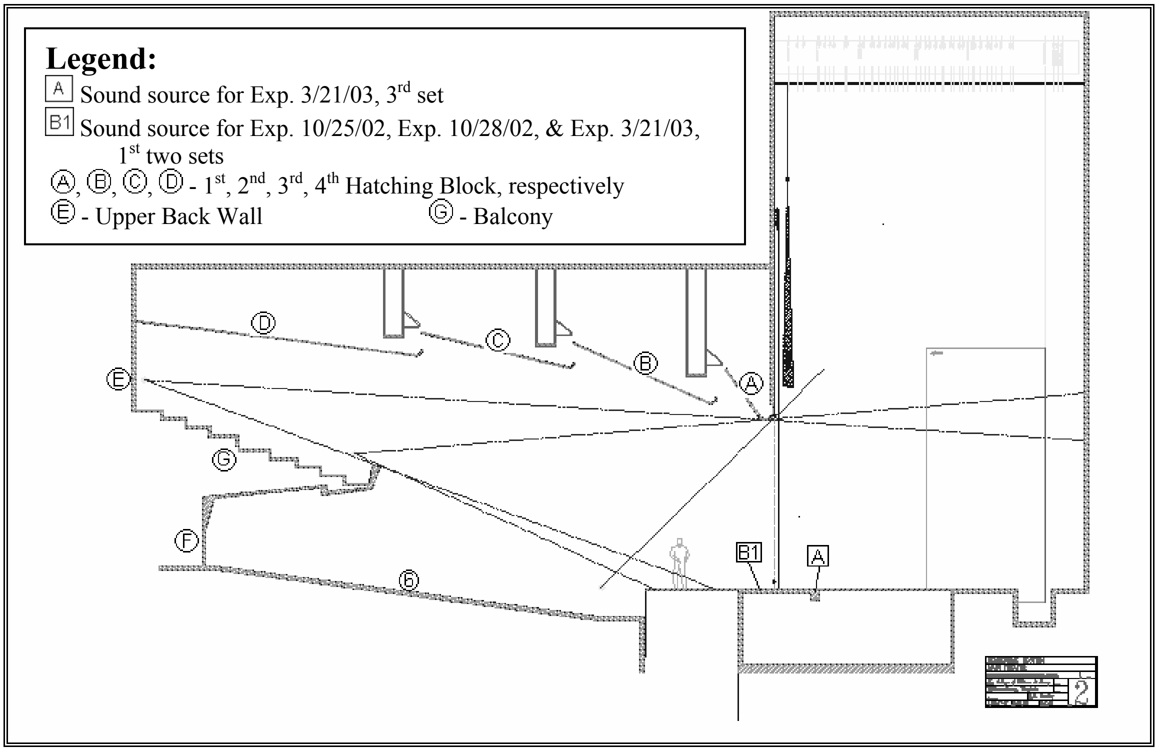


Figure 3 *– Side view of PBK Hall. The main stage is where the man is pictured, to his right are the two sound source points,* A *&* B1*. Above him are the 4 hatching blocks, which are large sound-reflecting panels, and to his left is the general seating (Point* 6*) and balcony (Point* G*). The lines drawn in are just artifacts of the AutoCAD blueprint that had showed certain distances.*

Description (Distance)	Measurement (Ft)	Measurement (m)	Time (s)	Time (rel. to direct sound) (s)
Direct - Source Point (SP) to Recording Point (RP)	22.1	6.73608	0.020412364	0.000
Indirect - SP to 2nd Hatching Block to RP	51	15.5448	0.047105455	**0.027**
SP to lower back wall to RP	112.6	34.32048	0.104001455	**0.084**
SP to balcony bottom lip to RP	77.3	23.56104	0.071397091	**0.051**

Table 3 *- Chart indicating the predicted time of a sound reflection using Figure 3 – Side View of PBK Hall. Measurements are made from the PBK diagram in feet and then converted to meters. The time of sound travel for a particular reflection* ("Time (s)") *is arrived by the equation* Time(s) = d / v, *where* D = distance traveled *and* v = speed of sound in air *(330m/s). Also,* "Time (rel. to direct sound) (s)" *indicates the predicted time after which sound from direct straight-line path arrives – in other words the predicted reflected time minus the direct source time.*

I returned a few days later (*Experiment 10/28/02)* to attempt to improve upon the first experiment by bringing along a solid 12"x12" plywood board as the sound source in an effort to create a more intense and compact impulse.

Point # / 6	Time of Impulse Peak (s)	Primary Echo Peak (s)	Time of First Reflection (s)
1	0.074	0.212	**0.138**
2	0.201	0.350	**0.149**
3	0.237	0.381	**0.144**
4	0.165	0.362	**0.197**
5	0.348	0.526	**0.178**
6	0.305	0.482	**0.177**

Table 4 *– Measurement of the highest point in the gradual rise in intensity before the more natural decay in* Experiment 10/28/02*'s plywood impulse test (see Figure 4). After seeing a strange gradual rise in sound intensity after the initial impulse before the eventual fall, I attempted to measure the highest point of that rise ("Primary Echo Peak") to see if it correlated to a specific time interval, but unfortunately there was no noticeable correlation. The six points in the left-most column correspond to the numbered points in Figure 1. The 2nd column is the time of the impulse peak in each clip, the 3rd column the time of the "Primary Echo Peak," the 4th column in bold is the time interval between the two ("Time of First Reflection"). Points 1, 2 & 5 being parallel recording points* should *have a similar time, as should Points 3, 4 and 6, Unfortunately, there is no such correlation.*

The plywood experiment was unfortunately unsuccessful in terms of a tight impulse generated. There was an observed sharp spike about 20ms following the initial impulse, but it is difficult to prove either way whether that sharp secondary spike (which is much louder than any impulses following, and was not observed in the first experiment) is indeed a sound reflection, possibly from the pit floor (which was raised for that experiment but not for the others), or simply another part of the same plywood board caused by an imperfect hitting of the plywood board to the floor (i.e. two parts of the board hitting each other, one part hitting first, followed shortly after the first). However, what was interesting at first observation was that all of the samples seemed to follow an overall pattern of an impulse followed by a sharp decline, then a gradual buildup to a peak, followed by a linear decay (for some examples see Figure 4 a/b/c). Upon following up, however, there was not any useable correlation, as measurements of the time between primary impulse and final swell did not correlate with each other (a correlation between Points 1 & 2, and Points 3 & 4 could be expected based on the idea that they are symmetrical to each other, but they do not have a similar value - see Table 4, and Figure 1).

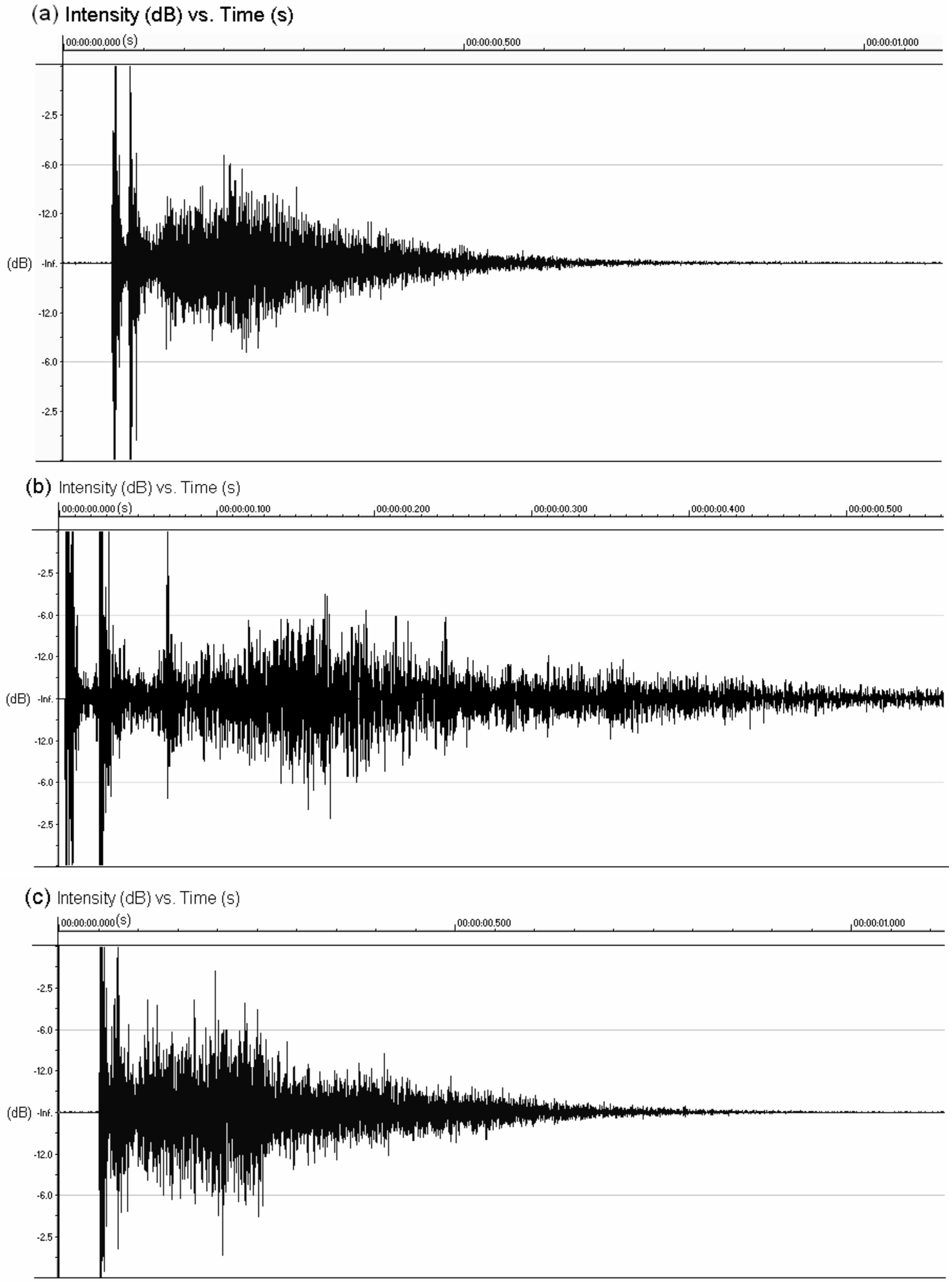


Figure 4 - *Sound intensity vs. time recordings from* Experiment 10/28/02 *(similar to* Experiment 10/25/02 *graphs in* Figure 2*), showing an initial sound impulse followed by subsequent sound peaks. These peaks correspond to large reflections of the initial impulse by a uniform surface. The sound source this time is a 12"X 12" piece of plywood thrown into the floor at point B1 (see Figure 1) by hand. They are recorded from (a) Point 1, (b) Point 2, and (c) Point 3 (see Figure 1). Note the double initial impulses in all of the graphs, which could be either the first reflected peak* or *the plywood hitting the floor imperfectly, causing two parts of the plywood to hit the floor at different times.*

Recording Type	Time of Impulse Peak (s)	Primary Echo Peak (s)	Time of First Reflection (s)
1000Hz 1ms	17.232	17.259	**0.027**
1000Hz 5ms	24.236	24.260	**0.024**
Noise 1ms	31.232	31.260	**0.028**
Noise 5ms	38.235	38.264	**0.029**

Table 5 *– Experiment 3/21/03: Approximate measure of primary echo after initial impulse. Four impulses were played from a computer-generated WAV file played from on CD (and recorded) in the experiment, a 1000Hz 1ms & 5ms impulse, and a 1ms & 5ms impulse of random noise. The* Time of Impulse Peak *is the time in the clip in which the impulse begins (for ease of calculations, the minute count of the time is not given) , the* Primary Echo Peak *is the peak of the 1st echo (estimated by looking at the raw amplitudinal graphs of the impulses), and the* Time of First Reflection *(in bold) is the time between the two peaks. Similar to earlier impulse experiments, except: (pro) a more precise peak, and (con) less intensity of impulse, therefore less-than-ideally strong reflections.*

Based on the previous experiments, I returned once again to PBK (*Experiment 3/21/03)* conduct a final impulse experiment. This time the sound source was a large Sony boombox of considerable volume and adequate fidelity, and a Compact Disc which contained twelve tracks - each track a computer-generated 10-second-long sound wave followed by 7 seconds of silence.

I took three sets of data, but because I wanted to do a full FFT analysis of multiple frequencies for the later EDT measurement, and for that EDT measurement due to the extremely time-consuming task of inputting data from the FFT program to the Excel Spreadsheet, I decided to analyze only Clip One.

The computer-generated impulses were not as successful as I had wished them to be. While successfully precise in its initial duration, viewing the raw amplitudes of the recording reveals that a) the speaker was not able to completely reproduce the cessation of the impulse, and b) the short impulse was not of a sufficient intensity to be able to easily record subsequent reflections (see Figure 5 a/b/c/d – note: for ease of reading the graph the time has been reset to close to 0 near the initial impulse). Nonetheless, I went ahead and used the same techniques as in *Experiment 10/28/02* (see Table 4) to visually approximate a reflective peak, with the results

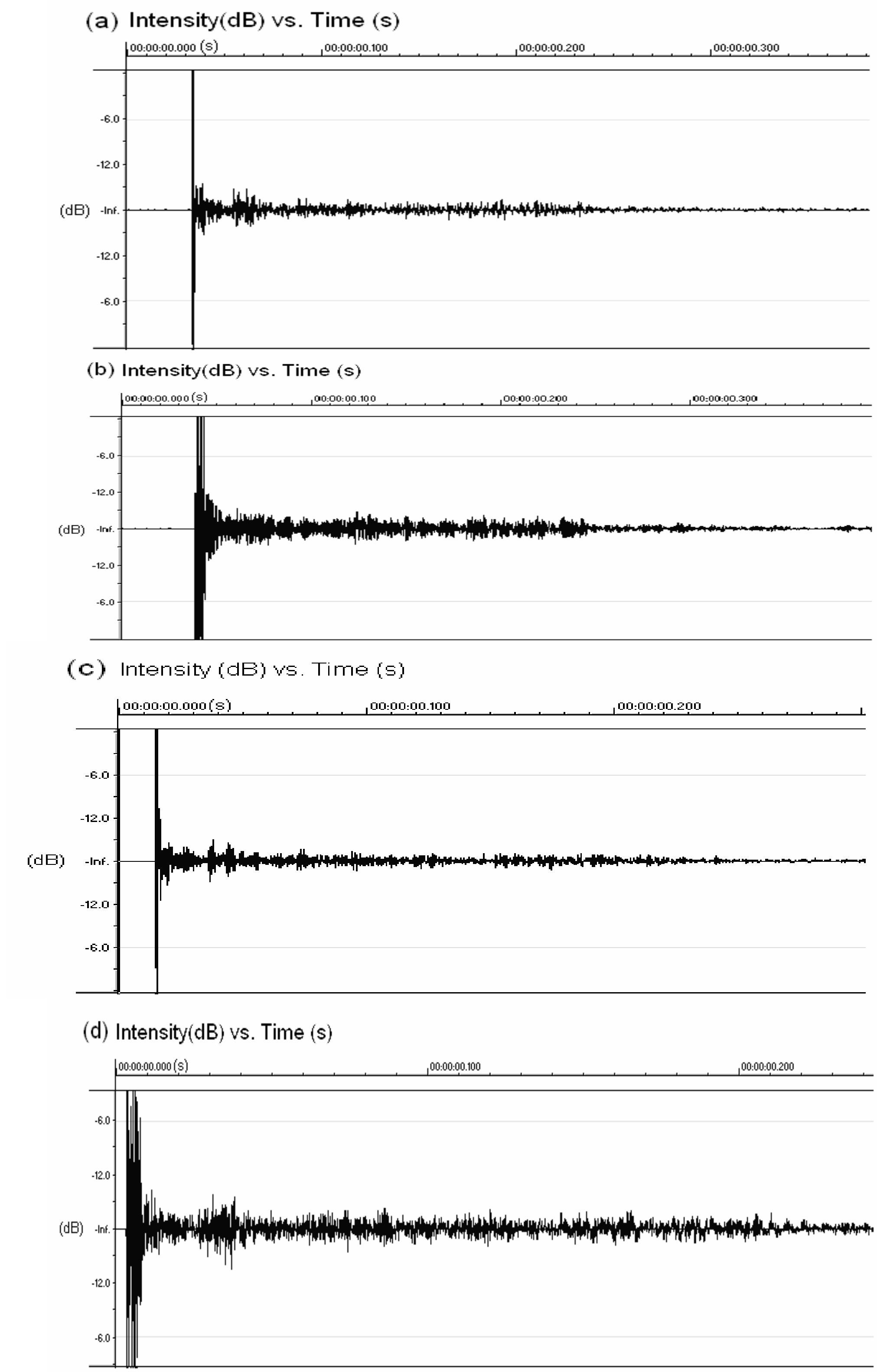


Figure 5: *Sound intensity vs. time recordings from* Experiment 3/21/03 *showing an initial sound impulse followed by subsequent sound peaks. Following graphs are for (a) 1000Hz for 1ms, (b) 1000Hz for 5ms, (c) randomly generated noise for 1ms, and (d) randomly generated noise for 5ms. Impulse time length is more precise due to computer-generated wave-form played back from CD, but the weak reflected peaks are attributed to an insufficiently loud impulse. These graphs were used for Table 5.*

given in Table 5. All four recordings measure a primary sound peak between 24 and 29ms, meaning the main reflections seem to be coming from the path of the Sound Point to the 2nd Hatching Block to the Recording Point (see Table 5 & Table 3).

D. Standing Wave Interference Experiments

Recording Point	Intensity (dB)
1	66
2	57
3	60
4	64
5	66
6	56

Table 6 *- Measurement of Sound Intensity using a handheld sound meter at several Points (see Figure 1 for corresponding Points).*

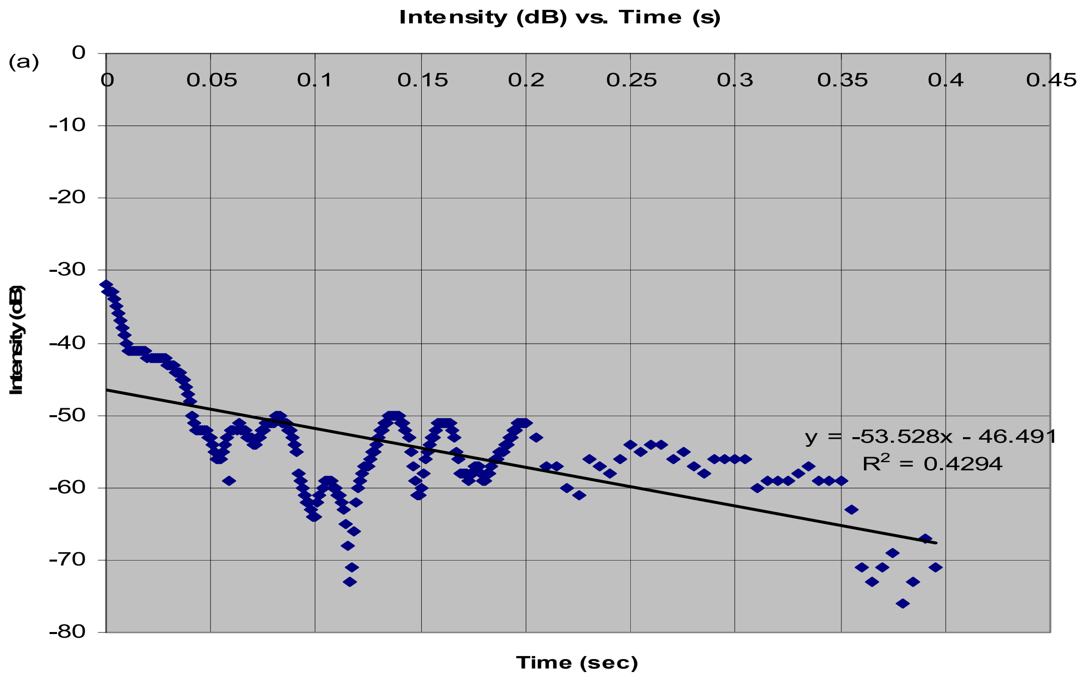
The goal of this experiment (done in *Experiment 3/28/02*) was to check for signs of constructive/deconstructive interference in the hall, by playing a constant sound from the stage (a ~1KHz sine wave generated by a frequency generator and connected to a speaker at Point B1) and using the sound intensity meter at various points about PBK to measure the variation. Intensity levels (in dB) at *Points 1-6* were recorded, as well as general levels moving from left to right in the orchestra section, also up and down, as well as up in the balcony section (see Table 6).

This test to see what the level of interference of sound due to standing waves created by solidly reflecting surfaces in PBK was quite revealing. Playing a 1000Hz frequency using the frequency generator and recording the sound intensity at *Points 1-6* (see Table 6), there was an amazingly large 10dB different between different points in the Hall, especially the large 9dB different between *Points 1* & *2*, which are symmetrical locations relative to the sound source, so barring interference should have a similar sound intensity. Also, walking along row N (the first complete row included in all three columns of seats on the PBK ground floor and directly beneath the lower lip of the balcony, see Figures 1&3), I audibly perceived bands of constructive and deconstructive interference, producing a “wa-wa” effect, and with my hand-held sound meter recorded a change in intensity level along that row (despite its near equidistance from the sound source) of *12dBs* (from 52dB to 65dB). Also, in the same row, there was vertical interference as well – there was a perceivable change in intensity (same “wa-wa” effect) as I moved my head from just above the seat tops to a position approximately 1.6m above the floor (the position of my ears from the floor when standing upright). Similarly, in the balcony a quick walk along the balcony revealed the same effect, but with less variation. Also, a quick switch to 200Hz yielded less audibly noticeable interference than with the 1000Hz sound source, but it was still present.

E. Measurements of EDT using Fourier Analysis.

The first attempted measurement of EDT using Fourier Analysis was for *Experiment 10/28/02,* when a sine wave was played (produced by the wave generator, approximately 1000Hz, later analyzed by the WAV analyzer program to be 939Hz), allowing it to saturate the room, and then cutting it off to measure its decay. This process was repeated to check for consistency. The analysis of those recorded sine waves is when the Fast Fourier Transform (FFT) component of the WAV-editing program was first tested. Because of the painstaking (and time-consuming) effort involved in gathering Fourier-analyzed intensity data and getting into graph-able form, from this point on all EDT data sets were recorded at *Point 6* (see Figure 1), the center seat in row H (in the middle of the orchestra seats).

The recordings of the sine waves were a good start, but several problems in this experiment became evident. Unfortunately, the generated sound was not intense enough to begin with (with a maximum amplitude of -32dB, the noise level threshold of my recording device



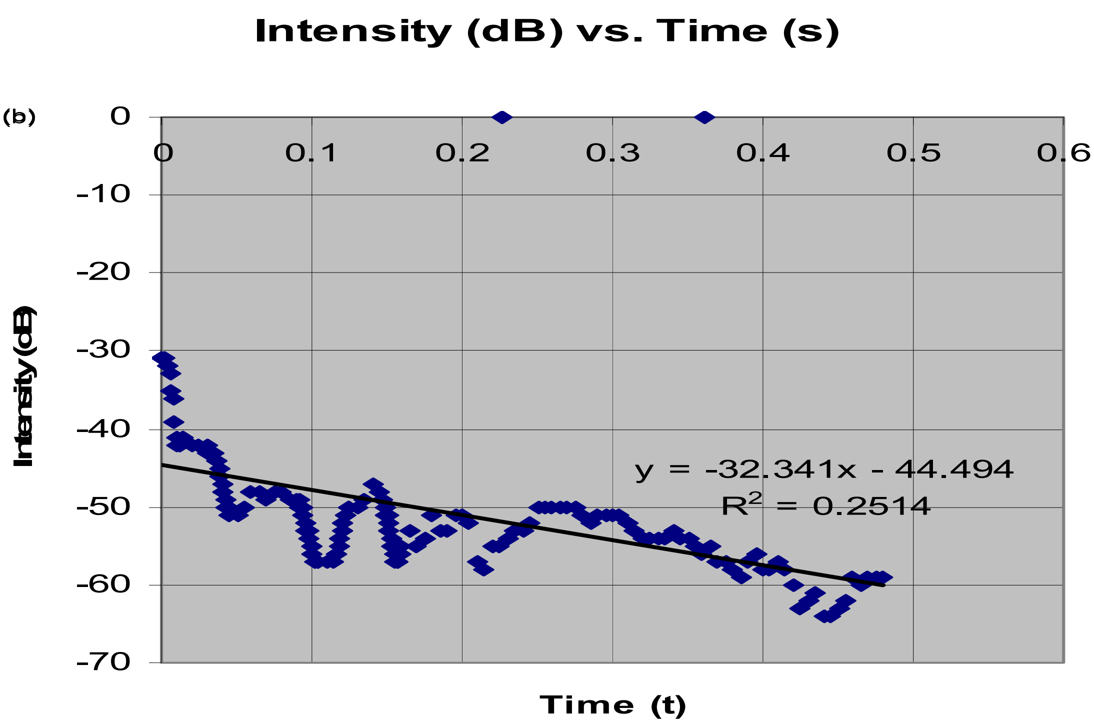


Figure 6 *– Experiment 10/28/02: Use of Fourier Analysis to measure sound intensity vs. time for a sustained sine wave allowed to saturate a room, then instantaneously cut off.* Time = 0 *is the instant the sound cuts off, as seen by looking at the amplitude of the recording. For the analysis of the two data sets* (a) *&* (b*) (the second identical test to check for consistency*), *the WAV editor used a Fast Fourier Transform (FFT) Blackman-Harris algorithm of 2048 samples to look at the sound intensity of the specific spectrum of the main frequency produced by the sine-wave generator (about 939Hz), and the results were fed into a Microsoft Excel spreadsheet and subsequently graphed. Ideally there should be a linear decay, but both measurements* (a) *&* (b) *reveal many exceptions to the linear decay. The equation on the graph is best-fit line made by Excel. The* R^2 *value of the line fit (measure of accuracy in fitting the curve) is quite low (0 means 0% accurate,1 = 100% accurate),which could explain the very different slopes between the two data slopes, both of which should be hypothetically similar. Also, around -65dB is the threshold of the maximum sensitivity of the recording device, after which point data is random and unreliable.*

being about -70dB), resulting in an unreliable sign of decay that quickly (within about ~300-400ms) reached the noise level threshold (see Figure 6 a/b). Also the physical act of pulling out the plug for the signal generator created a popping sound, thus possibly marring the sine wave at that instant as well.

Therefore I decided for the next experiment (*Experiment 3/21/03*) that I would use my WAV-editing program to construct a WAV file containing a number of sine waves that would play long enough to saturate the room and then would suddenly cease, allowing for "no human error" recording of decay (EDT). This WAV file would be burned to a CD, and played a larger wattage high-fidelity music system equipped with a CD player.

The results from this improved-intensity experiment were far better than with *Experiment 10/28/02*. On some of the graphs, upwards of 15dBs were reached, resulting in a decent linear fit (see Figure 7 and Figure 8 – note – all these graphs use FFT @ 2048 samples). On the graphical analysis of the set frequencies, the error (the R^2 value given in the graph, where $R^2 = 1$ means it is a perfect fit, $R^2 = 0$ means it does not fit at all) is much less than the previous attempt (see Figure 5 a/b). Even the Fourier analysis on the noise sample (see Figure 8), while understandably not having a high R^2 value, is very close in terms of its slope (see Table 8 for extracted EDT times). However, there is definitely unevenness in every single graph, indicating an unevenness of sound decay (Note – in the graphs, the point 0 has been designated as the time, in seconds, at which the sound from the direct path has began to cease being recorded by the recording device, which one can tell by looking at the raw waveform).

At this point something interesting about the FFT method must be mentioned. FFT takes an average number of samples before and after a certain point, and depending on the sample rate (in my case I was using 256 & 2048), ~26ms (2048/48KHz divided by 2) or ~5ms (256/48KHz

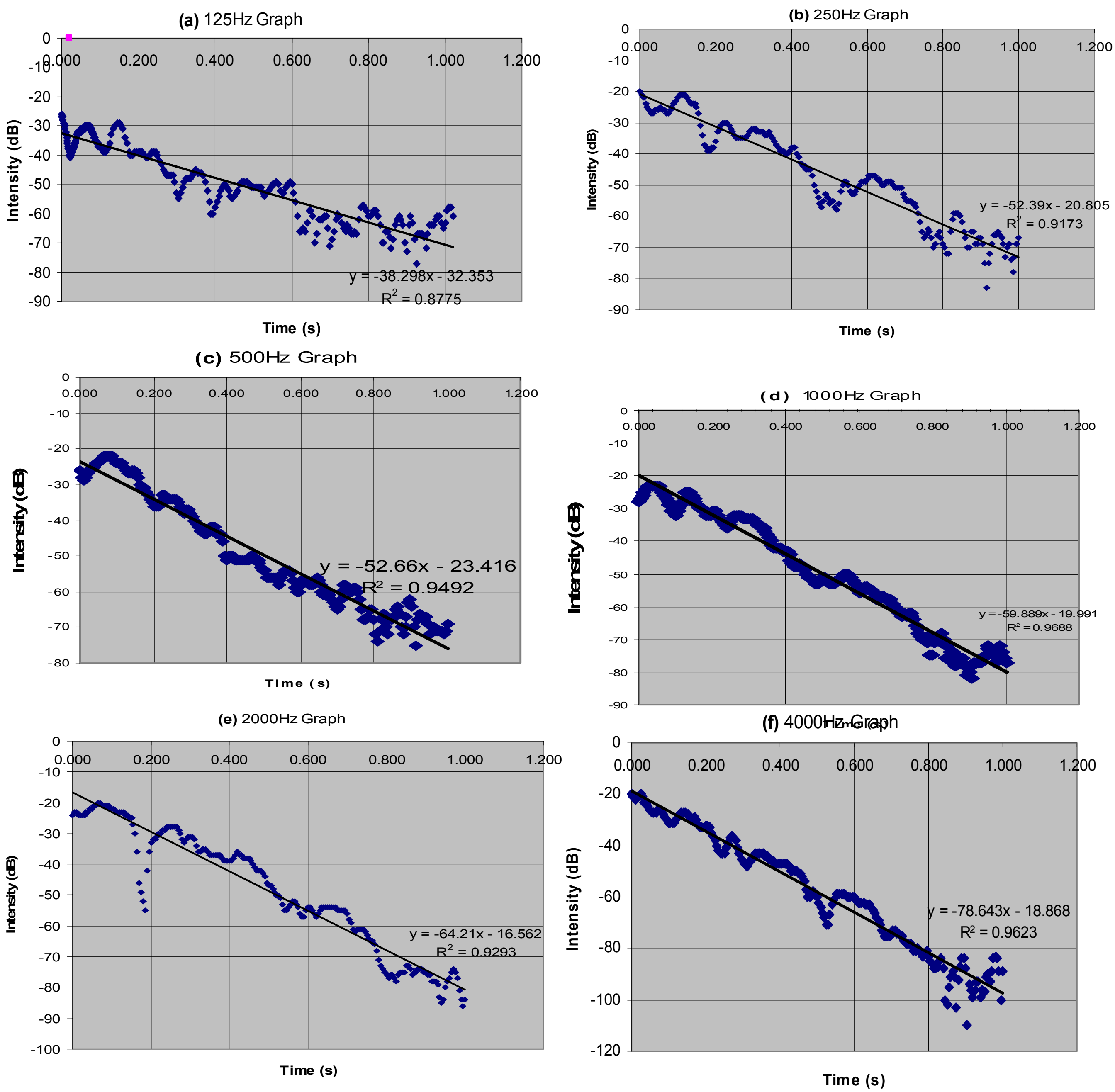


Figure 7 *– Experiment 3/21/03: FFT measurements of the decay rate of intensity over time from Clip 1, in which computer-generated sine-waves of various frequencies were allowed to saturate the Hall and then cut off, after which the decay was measured. The Sound Source was recorded at Point B1, and the Recording Source at Point 6 (See Figure 1&3). Frequencies measured were (a) 125Hz, (b) 250Hz, (c) 500Hz, (d) 1000Hz, (e) 2000Hz, and (f) 4000Hz. The equation describes the best-fit linear equation predicted by exponential decay of sound (linear in this case because the dB scale is logarithmic), the* R^2 *value indicates the great precision of the fit (0 = 0% accurate, 1=100% accurate). Note the overall accuracy of the linear slope, but also the smaller uneven shape of the line, indicating less than ideal decay and possible reflections or constructive/deconstructive interference. More accurate results than Experiment 10/28/02, because the intensity started at a much higher level this time before dropping to noise threshold, which is around -70dB. Also note that the apexes and nadirs of each frequency do not quite correspond with each other, which makes matching peaks with reflective surfaces difficult correlate using Fourier Analysis.*

Intensity (dB)

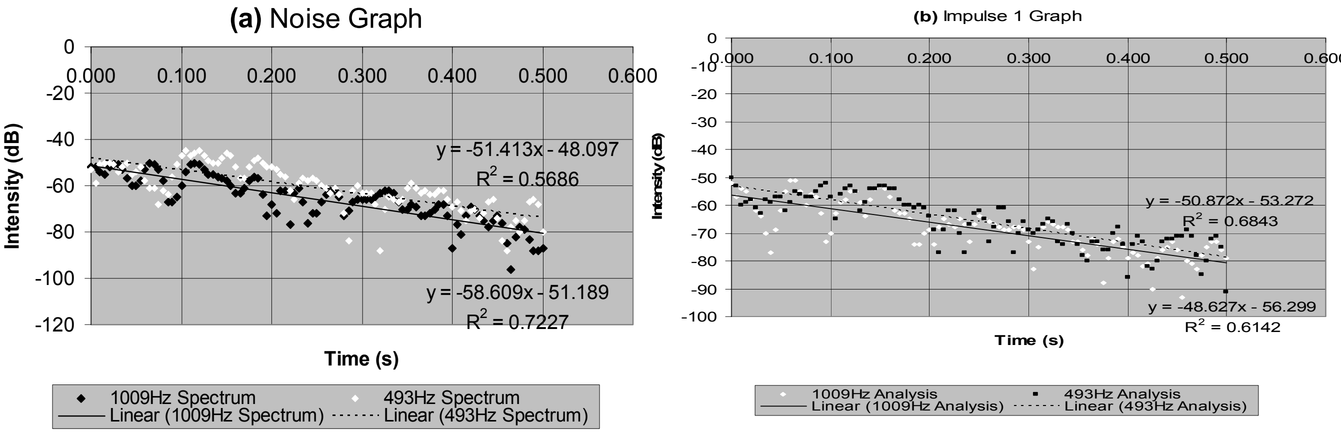


Figure 8 *- Testing the FFT algorithm by taking **(a)** noise and **(b)** impulse intensity levels and performing a Fourier analysis on it and focusing in on specific bandwidths of 500Hz & 1000Hz. While the* R^2 *value indicates quite a point spread from the line, the important thing to note is the slope of three out of the four slopes correlates closely with data taken from Figure 7 (compare:-51.413 from noise with -52.66 from Figure 7's 500Hz slope, -58.609 from noise with -59.889 from Figure 7's 1000Hz slope, -50.872 from impulse with -52.66 for 500Hz, and -48.627 from impulse with -59.889 for 1000Hz). The point: FFT works.*

divided by 2) before the actual intensity drop will already begin to appear to fall. To test this theory I focused in on the 500Hz and 1000Hz bands (so chosen because looking at their waves, they are the purest sine waves, with the least distortion – see Figure 10 – this low distortion is important, as higher-sample FFT will filter out distortion at the price of inaccuracy of time, but lower-sample FFT will not filter out distortion but will be more accurate time-wise), and did a Fourier analysis backwards 200ms from the point that the sound was cut off as well, to confirm that the sustained decibel levels were completely stable (as they were shown to be – see Figures 9a/b).

Another realization that came with this was that in terms of time accuracy of the FFT's, less was better. Therefore redoing again the 500Hz and 1000Hz bands (chosen because of their band purity, see explanation in previous paragraph) this time in 256 samples as oppose to 2048 samples, the lag time between reality and what was shown on the FFT graph disappeared. Note, however, that the time accuracy comes at a cost – the R^2 value of the 256 sample FFT graphs is higher.

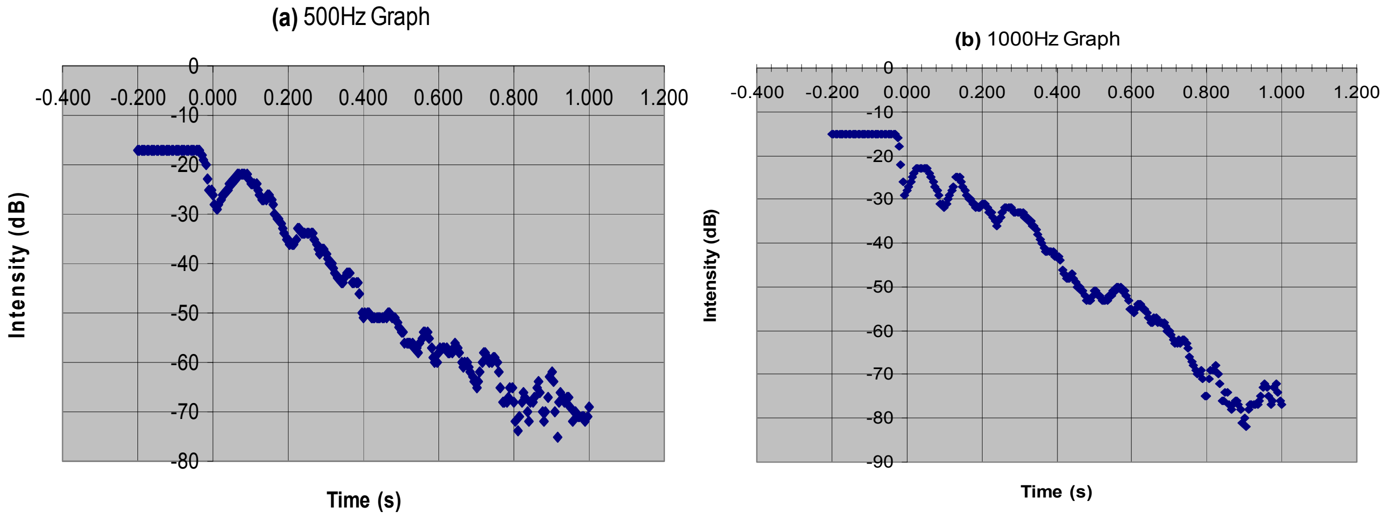


Figure 9 *– Experiment 3/21/03: Consistency of FFTs in measuring stable sine waves previous to cut-off. To validate that the erratic non-linearly decaying sound intensity data observed in Figure 7 using the FFT method was not caused by inherent recording error, the time (200ms) previous to the instant the sound was cut off was measured using the same original data. As seen in both the (a) 500Hz and the (b) 1000Hz graph, the intensity is constant throughout the saturation time, helping rule out instrument error as a possible cause for the non-linearity. Note how for the FFT, the intensity is actually shown to be lower* before *the observed point of sound cutoff. This is because FFT takes the average of a number of samples before and after the observed point in time (in this case 2048 samples, 1024 before, 1024 after). Therefore there should be a drop in intensity observed by this FFT at about 21.3ms (1024samples / 48,000samples/sec). And indeed, in the (a) 500Hz test the sound was observed to begin dropping ~30ms from the actual drop, and for the (b) 1000Hz test ~30ms, just as predicted.*

Also, between *Point 6* and the Clip One source I have plotted out a few possible points of reflection (see Table 7), and indeed comparing the two (theoretical points of reflection with actual recorded changes in intensity) there are enough correlations to show that the reasons for the unwanted, uneven decay come from reflections off the surfaces mentioned (see Table 7).

One must note, however, that perhaps the weakest link in the entire project is the rough approximation of the predicted reflected sound distances and therefore time delays, which is based on a very loose method of opening up the AutoCAD reader, drawing lines from the point source and its path to the eventual recording source, applying the angular reflections not by rulers and protractors, but by eye. For this reason alone I believe that a more solid theoretical conclusion could be reached about the specific sources of acoustical undesireability, matching the experimental results, if a more precise modeling system could have been used. This problem

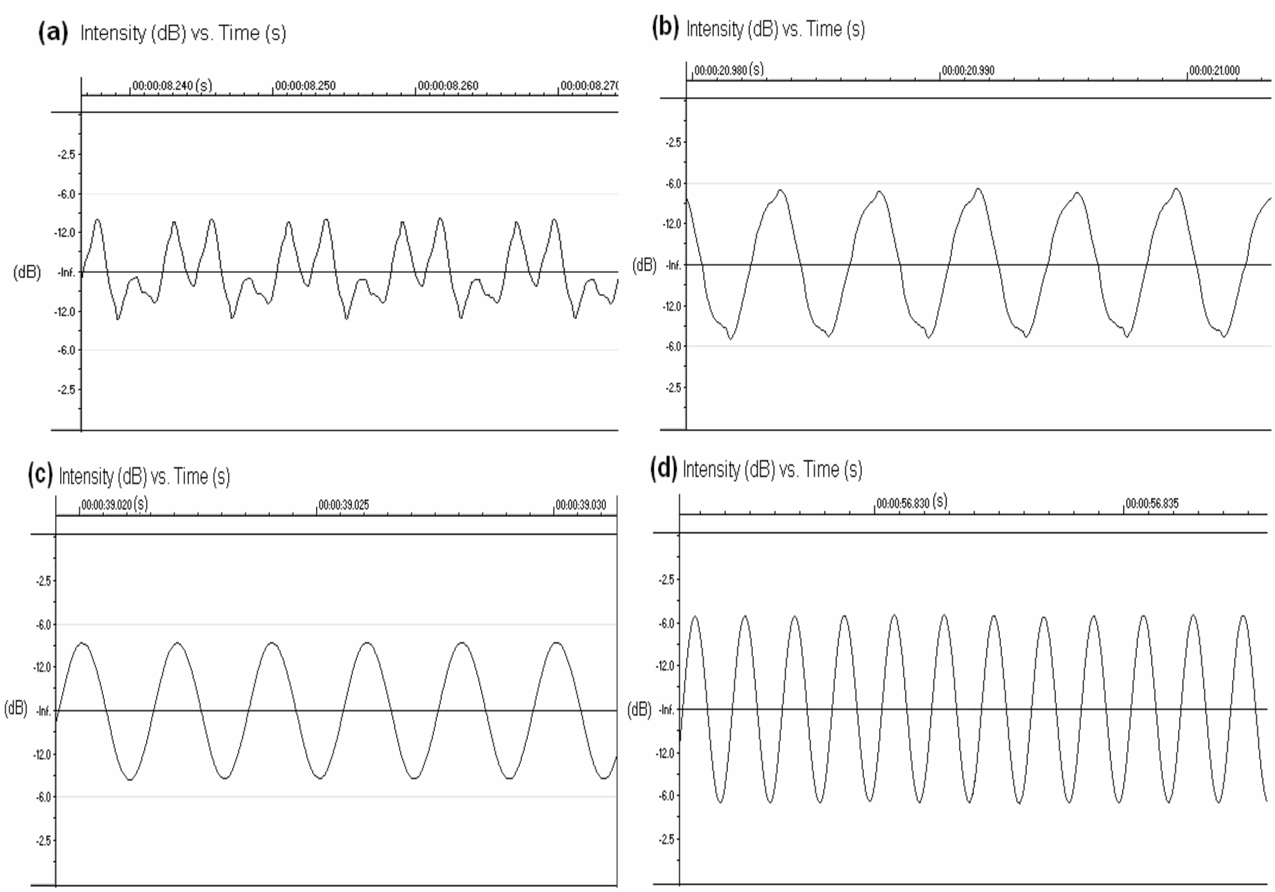
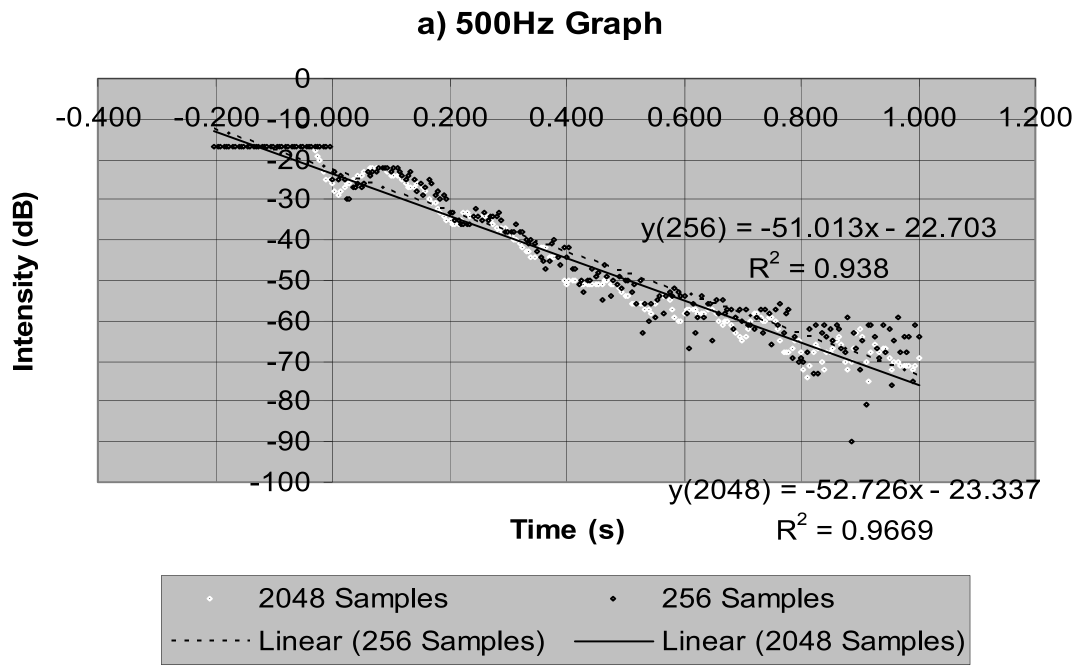


***Figure 10** – Observed distortion of pure sine waves after having been output through the sound source (speakers). The above are recordings from Clip 1 in Experiment 3/21/03 – while of varying frequency, all sources were computer-generated to be pure sign waves, and the graphs show the level of distortion exacted for **(a)** 125Hz, **(b)** 250Hz, **(c)** 500Hz, and **(d)** 1000Hz. The most preserved sine waves were at 500Hz and 1000Hz, whereas 125Hz and 250Hz had quite noticeable (both by looking at the actual wave and by hearing the overtones by ear) distortion, most probably caused by the inability of the speaker system used to handle the high intensities of vibrations required to perfectly reproduce the lower frequency sine waves.*

became glaringly obvious when I was comparing Table 3 to Table 7, both which share a number of measurements, but with completely different values for said measurements. It is indeed disconcerting, and one would like to reconfirm my approximations, but unfortunately the trial version of the CADView program I was using has expired recently, and I cannot view the blueprints and re-measure. Therefore the correlary data cannot be relied upon as accurate.

Finally, I calculated the average Early Decay Time (EDT) using the 125/250/500/1000/2000/4000Hz fits (see Table 8), and found the ***EDT = 1.1sec***.



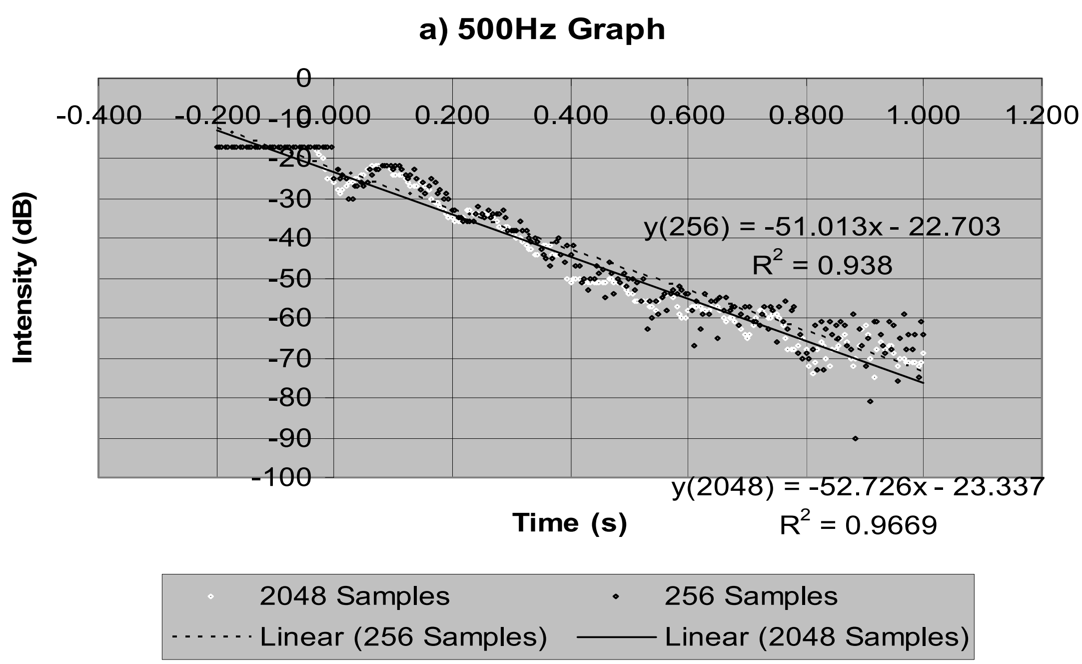


Figure 11 *– Intensity vs. Time graph comparing different FFT sample sizes at a) 500Hz and b)1000Hz. The graphs show how a 256 sample FFT is more accurate time-wise (intensity drops down at t = 0) than a 2048 sample FFT, but at the cost of overall accuracy – the 2048 sample FFT has a higher* R^2 *value (less error of line fit) than the 256 sample FFT.*

Description (Distance)	Measurement (Ft)	Measurement (m)	Time (s)	Time (rel. to direct sound)
Vertical ("Side-view") Measurements				
Direct - Source Point (SP) to Recording Point (RP)	41.70833333	12.7127	0.038523	**0**
Indirect - SP to 2nd Hatching Block to RP	63.38541667	19.319875	0.058545	**0.020021742**
SP to 2nd Hatching Block to Balcony to RP	78	23.7744	0.072044	**0.033520303**
SP to 2nd Hatching Block Front Lip to RP	65.75	20.0406	0.060729	**0.022205758**
SP to Balcony Bottom Lip to RC	59.5	18.1356	0.054956	**0.01643303**
SP to Lower Back Wall to RP	91.6	27.91968	0.084605	**0.046081758**
SP to Underside of Balcony to Back Wall to RP	93.8	28.59024	0.086637	**0.048113758**
SP to 1st Hatching Block Front Lip to RP	65.6	19.99488	0.060591	**0.022067212**
Horizontal ("Bird's Eye View") Measurements				
Direct - SP to RP	35.48	10.814304	0.032771	**0**
SP to Side Wall to Back Wall to RP	117	35.6616	0.108065	**0.075294836**
SP to Back Wall to RP (no angles - straight line)	94.5	28.8036	0.087284	**0.054513018**
SP to Curved Side Wall Lip (near stairs) to RP	57	17.3736	0.052647	**0.019876655**

***Table** 7 – Estimated time of sound reflections following the direct source, derived from approximations of distances of likely sound-reflecting paths. The "Side-View" measurements are based on an AutoCAD version of Figure 3, the "Bird's Eye View" based on Figure 1; the Source Point (SP) refers to Point B1, the Recording Point (RP) refers to Point 6 (see Figures 1 & 3). Due to nature of AutoCAD viewer, estimations were first made in feet (2^{nd} column), then converted to meters (3^{rd} column). Then distances were converted to time taken to travel said distance using equation* time = distance / velocity *(4^{th} column). Finally, the direct SP to RP travel time was subtracted from the travel time of each path, giving the estimated time after the direct sound that a sound reflection would occur (5^{th} column, in bold).*

V. Interpretation of Results

Of the three impulse tests (to attempt to directly measure, through raw amplitudinal analysis, sources of unwanted reflective sources), each improved in one way or another. The sound intensity for the first test (in *Experiment 10/25/02*, using a notebook) was insufficient, and the impulse length was longer than desired. The second test was much louder (and thus measurable), but the longer-than-desired impulse length remained, along with an anomalous second spike. The clinical accuracy through the computer-generated impulses in the final experiment, while much improving the brevity of the impulse, unfortunately yielded insufficient intensity to be accurately measured. Although in the final test enough data was gathered to attempt matching it with predicted reflection peaks based on a geometric modeling, the insufficient intensity of the impulse made it a step back in progress, unfortunately.

Frequency (Hz)	*125*	*250*	*500*	*1000*	*2000*	*4000*	
R^2 Value of Best-Fit Curve	0.8775	0.9173	0.9492	0.9688	0.9293	0.9623	
Time of 10dB Decay	0.252	0.21	0.19	0.167	0.156	0.127	
EDT (s)	1.512	1.26	1.14	1.002	0.936	0.762	
Average EDT (s)							**1.102**

Table 8 *– Experiment 3/21/03: Early Decay Time (EDT) measured, based on data shown in Figure 7.* EDT is defined as six times the length of time that it takes for a sound to decay 10dB after the instant the sound source ceases. *For each frequency, the* R^2 *value indicates the accuracy of the line fit, which in this case is linear (*$R^2 = 0$ *indicates 0% correlation,* $R^2 = 1$ *indicates every point falls on the line). The 10dB decay was measured based on the line-fit equation (see Figure 7), not by looking at the measurements of intensity versus time and seeing when it dropped 10dB, the equation being more reliable (high* R^2 *value of enough equations) than the measurements with the problems of uneven decay as has been described earlier. The Average EDT averages the other frequencies' EDT and gives the final measurement for EDT. Notice the trend of the EDT falling as the frequency rises.*

As mentioned earlier in the **Data Collection and Analysis** section, the section that could be improved upon the most was in the geometric approximations made in predicting sound reflections from various surfaces in PBK Hall. The way I calculated the distances was by using an AutoCAD viewer program that would allow me to do a line trace of a path in scale with the distances in PBK. However, in terms of reflection off various surfaces, the angles were done "by eye" and not protractor, and therefore the distances (and subsequent time expectations for various surfaces) have a larger error than is desirable. Compare, for example, Table 3 vs. Table 7, both created independently and both sharing sound reflections that should have equal value, but do not. Also take into account the inaccuracies of the PBK AutoCAD blueprints themselves – I compared some distances of ceilings heights given in the blueprints with physical measurements of these values, and I found an error of approximately 1ft for measurements from 21ft to 35ft. These inaccuracies have made pinpointing the specific sources of each undesirable reflection more difficult. If this project were to have been repeated again a more accurate modeling system would have been pursued.

From *Experiment 10/28/02* I was thoroughly appalled at the extent of interference and variation of sound intensity, especially underneath the balcony area as I had observed in Table 6. Also, that was so much variation among even supposedly symmetrical points is cause for acoustical concern.

Figure 8 was a test of the ability of the Fourier Transform to make sense out of the "chaos" of both noise and a quick impulse. While the R^2 value for the line fit was understandably very low, that the slope was close to the measured value for each frequency was a good sign that the Fourier Transform was working properly.

The wave forms shown in Figure 10 reveal the distortion problems with the particular equipment used, and in what ranges.

The measurement of the Early Decay Time (EDT) = ~1.1 seconds reveals a *very* dry hall, considering the definition for a "dry" concert hall was said to be around RT = ~1.3 seconds, and EDT given to be typical about 0.2 seconds longer than that. While a few opera houses have been described to have good acoustics and yet also low reverberation time (Paris' Opera Garnier), in general an EDT of about 1.5~2.0 seconds seems to be preferred (see Table 9 for a listing of EDTs for a few world-class concert and opera halls next to W&M's Phi Beta Kappa Hall).

Category	City	Name of Hall	EDT (s)
"Superior"	Amersterdam	Concertgebouw	2.6
	Boston	Symphony Hall	2.4
	Vienna	Gr. Musikvereinssaal	3.0
"Excellent"	Berlin	Konzerthaus	2.4
"Fair"	London	Royal Albert Hall	2.6
Unclassified	New York	Avery Fisher Hall	1.8
	New York	Metropolitan Opera House	2.0
	Paris	Opera Garnier	1.2
	Williamsburg	**Phi Beta Kappa Hall**	**1.1**

Table 9 *– Comparisons of EDT of various concert halls and opera houses with William & Mary's Phi Beta Kappa Hall. Category ranking taken from a subjective rating method, and EDT from previously recorded EDT times (Beranek 1996, 51-61)*

VI. Conclusions

From all of the above the following general statements can be made:

The addition of a shell during concert performances (a technique indeed employed by the various large ensembles) could provide more sound within the initial time delay gap, and perhaps less sound to be reflected by the walls.

The inherent design of PBK Hall is indeed for short reverberation time, as it was designed primarily for theatre, in which clear understanding of speech requires a more direct sound environment. Any suggestions or improvements will not change this fact, which is sad for concert hall acoustics as according to Beranek healthy reverberation time seems to be one of the biggest desirable factors in a concert hall.

The addition of specially designed "sound diffusion" panels in key areas could greatly reduce the uneven sound decay that contributes to the unenjoyable PBK acoustics. Instead of sound absorption panels (which, since PBK absorbs sound so well already, would not be desirable), these panels would need to consist of a randomly-angled mosaic of many different-shaped surfaces. The random angles would serve to diffuse the sound (and thus minimize the peaks observed when sound would reflect directly from that surface), and the different sizes would serve to deflect different wavelengths of sound (as higher frequencies have smaller wavelengths and thus interact with smaller surfaces, and lower frequencies have longer wavelength and interact with larger surfaces). These key areas could include: a) the entirety of the back wall, the b) lower and upper lip of the balcony, c) the upper back wall (near the balcony), and d) the lip of the second hatching. The fan-shape design of the Hall seems to allow the side walls to be more or less untouched – as long as the previously mentioned surfaces are covered with diffusive material, a most costly side wall covering could be unnecessary.

The issue of the standing wave, with constructive and deconstructive interference changing by 10 dB or more the sound intensity chair-by-chair, while probably caused by the same surfaces described above, was not explored fully. In the future more could be done to attempt to smooth out those particular inconsistencies of the Hall. Since oftentimes standing waves are caused by two parallel surfaces, perhaps the placing of diffusive paneling specifically on the hatching blocks (which are roughly parallel to the floor) and on the underside of the balcony would do much to destroy this constructive/deconstructive interference.

The area to follow up on the most would be to come up with a better method of modeling PBK Hall, perhaps writing a computer program that would simulate what was done "by hand," and thus be more accurate. Also, if more comprehensive tests were to be undertaken, a much easier way of extrapolating sound intensity from a WAV file must be found. The time-consuming manual method of analyzing a WAV file nearly point by point with a spectral analyzer forced this project to limit the amount of data analyzed.

In the end, PBK Hall is not ideal as a concert hall, nor was it designed to be a concert hall. However, with some slight (and not-so-expensive) modification, one could improve the acoustics, and raise its appeal at least slightly so that it does not remain the concert hall that concert musicians begrudgingly perform in out of no other alternative in this area.

References

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Appendix A

Example of one Data Table – The data behind Figure 9a

Clip One Data Analysis:
500Hz & 1000Hz Table

Note: High Noise level around 516Hz, so used Spectrum Analyzer to limit to levels on what it read to be 493Hz
Note: Fourier Analysis: Blackman-Harris Method, Left Channel, 2048 samples

Clip Section	500Hz (493Hz experimentally)
Section start time (s)	45.232
Section end time (s)	46.232

Data Points

Clip Time (s)	Section Time (s)	Loudness (dB)
45.032	-0.200	-17
45.037	-0.195	-17
45.042	-0.190	-17
45.047	-0.185	-17
45.052	-0.180	-17
45.057	-0.175	-17
45.062	-0.170	-17
45.067	-0.165	-17
45.072	-0.160	-17
45.077	-0.155	-17
45.082	-0.150	-17
45.087	-0.145	-17
45.092	-0.140	-17
45.097	-0.135	-17
45.102	-0.130	-17
45.107	-0.125	-17
45.112	-0.120	-17
45.117	-0.115	-17
45.122	-0.110	-17
45.127	-0.105	-17
45.132	-0.100	-17
45.137	-0.095	-17
45.142	-0.090	-17
45.147	-0.085	-17
45.152	-0.080	-17
45.157	-0.075	-17
45.162	-0.070	-17
45.167	-0.065	-17
45.172	-0.060	-17
45.177	-0.055	-17
45.182	-0.050	-17
45.187	-0.045	-17
45.192	-0.040	-17
45.197	-0.035	-17
45.202	-0.030	-18
45.207	-0.025	-19
45.212	-0.020	-20
45.217	-0.015	-23
45.222	-0.010	-25
45.227	-0.005	-25
45.232	0.000	-26
45.237	0.005	-28
45.242	0.010	-29
45.247	0.015	-28
45.252	0.020	-27
45.257	0.025	-26
45.262	0.030	-26
45.267	0.035	-25
45.272	0.040	-25
45.277	0.045	-24
45.282	0.050	-24
45.287	0.055	-23
45.292	0.060	-23
45.297	0.065	-22
45.302	0.070	-22
45.307	0.075	-22
45.312	0.080	-22
45.317	0.085	-22
45.322	0.090	-22
45.327	0.095	-23
45.332	0.100	-24
45.337	0.105	-24
45.342	0.110	-24
45.347	0.115	-24
45.352	0.120	-25
45.357	0.125	-26
45.362	0.130	-27
45.367	0.135	-27
45.372	0.140	-27
45.377	0.145	-26
45.382	0.150	-26
45.387	0.155	-27
45.392	0.160	-28
45.397	0.165	-30
45.402	0.170	-31
45.407	0.175	-31
45.412	0.180	-32
45.417	0.185	-33
45.422	0.190	-34
45.427	0.195	-35

45.432	0.200	-35	45.682	0.450	-51
45.437	0.205	-36	45.687	0.455	-51
45.442	0.210	-36	45.692	0.460	-51
45.447	0.215	-36	45.697	0.465	-50
45.452	0.220	-35	45.702	0.470	-50
45.457	0.225	-33	45.707	0.475	-51
45.462	0.230	-33	45.712	0.480	-51
45.467	0.235	-34	45.717	0.485	-51
45.472	0.240	-34	45.722	0.490	-52
45.477	0.245	-34	45.727	0.495	-53
45.482	0.250	-34	45.732	0.500	-54
45.487	0.255	-34	45.737	0.505	-54
45.492	0.260	-34	45.742	0.510	-56
45.497	0.265	-34	45.747	0.515	-56
45.502	0.270	-35	45.752	0.520	-56
45.507	0.275	-36	45.757	0.525	-56
45.512	0.280	-37	45.762	0.530	-56
45.517	0.285	-38	45.767	0.535	-57
45.522	0.290	-37	45.772	0.540	-57
45.527	0.295	-37	45.777	0.545	-58
45.532	0.300	-38	45.782	0.550	-56
45.537	0.305	-39	45.787	0.555	-55
45.542	0.310	-40	45.792	0.560	-54
45.547	0.315	-40	45.797	0.565	-54
45.552	0.320	-41	45.802	0.570	-54
45.557	0.325	-42	45.807	0.575	-55
45.562	0.330	-43	45.812	0.580	-57
45.567	0.335	-43	45.817	0.585	-59
45.572	0.340	-44	45.822	0.590	-60
45.577	0.345	-44	45.827	0.595	-60
45.582	0.350	-43	45.832	0.600	-58
45.587	0.355	-42	45.837	0.605	-57
45.592	0.360	-42	45.842	0.610	-57
45.597	0.365	-42	45.847	0.615	-57
45.602	0.370	-44	45.852	0.620	-57
45.607	0.375	-44	45.857	0.625	-58
45.612	0.380	-44	45.862	0.630	-58
45.617	0.385	-44	45.867	0.635	-58
45.622	0.390	-46	45.872	0.640	-57
45.627	0.395	-50	45.877	0.645	-56
45.632	0.400	-51	45.882	0.650	-57
45.637	0.405	-50	45.887	0.655	-58
45.642	0.410	-50	45.892	0.660	-60
45.647	0.415	-50	45.897	0.665	-61
45.652	0.420	-51	45.902	0.670	-60
45.657	0.425	-51	45.907	0.675	-60
45.662	0.430	-51	45.912	0.680	-61
45.667	0.435	-51	45.917	0.685	-62
45.672	0.440	-51	45.922	0.690	-63
45.677	0.445	-51	45.927	0.695	-64

45.932	0.700	-65
45.937	0.705	-64
45.942	0.710	-62
45.947	0.715	-60
45.952	0.720	-58
45.957	0.725	-58
45.962	0.730	-59
45.967	0.735	-60
45.972	0.740	-60
45.977	0.745	-59
45.982	0.750	-59
45.987	0.755	-60
45.992	0.760	-62
45.997	0.765	-65
46.002	0.770	-68
46.007	0.775	-68
46.012	0.780	-68
46.017	0.785	-67
46.022	0.790	-65
46.027	0.795	-65
46.032	0.800	-68
46.037	0.805	-72
46.042	0.810	-74
46.047	0.815	-71
46.052	0.820	-68
46.057	0.825	-66
46.062	0.830	-67
46.067	0.835	-70
46.072	0.840	-72
46.077	0.845	-68
46.082	0.850	-68
46.087	0.855	-67
46.092	0.860	-65
46.097	0.865	-64
46.102	0.870	-66
46.107	0.875	-70
46.112	0.880	-72
46.117	0.885	-70
46.122	0.890	-67
46.127	0.895	-63
46.132	0.900	-62
46.137	0.905	-64
46.142	0.910	-70
46.147	0.915	-75
46.152	0.920	-68
46.157	0.925	-66
46.162	0.930	-67
46.167	0.935	-68
46.172	0.940	-67
46.177	0.945	-67
46.182	0.950	-69
46.187	0.955	-72
46.192	0.960	-70
46.197	0.965	-70
46.202	0.970	-71
46.207	0.975	-71
46.212	0.980	-71
46.217	0.985	-71
46.222	0.990	-72
46.227	0.995	-71
46.232	1.000	-69