Chapter 4

Kinematics - Velocity and Acceleration

4.1 Purpose

In this lab, the relationship between position, velocity and acceleration will be explored. In this experiment, friction will be neglected. Constant (uniform) acceleration due to the force of gravity will be investigated.

4.2 Introduction

Note: For this experiment, you will write a complete (formal) lab report and hand it in at the next meeting of your lab section. This lab can not be your dropped grade for the semester.

Velocity and acceleration are changes of position and velocity with respect to time. In the language of calculus derivatives with respect to time:

$$\vec{v}(t) = \frac{\vec{x_2}(t_2) - \vec{x_1}(t_1)}{t_2 - t_1} = \frac{\Delta\vec{x}}{\Delta t} \qquad (= \frac{d\vec{x}}{dt})$$

$$\vec{a}(t) = \frac{\vec{v_2}(t_2) - \vec{v_1}(t_1)}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t} \qquad (= \frac{d\vec{v}}{dt})$$

where $\vec{x}(t_i)$ is the position vector at time t_i, $\Delta\vec{x}(t)$ represents the change in the position and Δt is the change in time. These definitions are for **average** velocity and acceleration. In the limit as Δt becomes small (approaches zero), we have the **instantaneous** velocity and acceleration.

For constant acceleration, these definitions can be used to find the two basic relations between distance, velocity, and the constant acceleration:

$$\vec{v} = \vec{v}_0 + \vec{a}t \tag{4.1}$$

$$\vec{x} = \vec{x_0} + \vec{v_0}t + \frac{1}{2}\vec{a}t^2 \tag{4.2}$$

where $\vec{x}$ is the position as a function of time, $\vec{v}$ is the velocity as a function of time, $\vec{v}_0$ is the initial velocity, $\vec{x}_0$ is the initial position, t is time and $\vec{a}$ is the constant acceleration. Notice that, for the special case of $\vec{a} = 0$, these equations reduce to the familiar $\vec{x} = \vec{x}_0 + \vec{v}_0 t$.

In the gravitational field near the earth, the acceleration in the vertical direction is constant and has a value of **g = 9.8 m/s^2 downward.** There is no acceleration in the horizontal direction for a free falling object. We can analyze the horizontal motion independently of the vertical motion. For the horizontal we have the simple case of x = $x_0 + v_{x0}t$. In the vertical, we have:

$$v_y = v_{y0} - gt$$

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

where we have chosen the positive y direction as vertically upward.

Finally, note about these relationships that:

- The general form of the position curve of a projectile is an inverted parabola. The maximum y value occurs when $v_y = 0$.
- The velocity is a straight line with a negative slope.
- The conservative nature of the gravitational force requires that the **speed** of the particle have the same value at the same y position going up and coming down. Of course, the direction of the vertical velocity component is reversed between the upward and downward portions of the trajectory.

4.3 Procedure

4.3.1 Motion of a Tossed Ball

In this part of the experiment, a motion detector will be used to collect distance, velocity, and acceleration data for a ball thrown upward. The ball and motion detector are shown in Figure 4.1.

4.3.2 Procedure

- Open the file 'ball_toss' with Capstone. Three graphs will be displayed: distance vs. time, velocity vs. time, and acceleration vs. time.
- Click 'record' to begin data collection. Toss the ball straight upward above the motion detector and let it fall back toward the motion detector. This step may require some practice. Hold the ball directly above and about 20 cm from the motion detector. (You

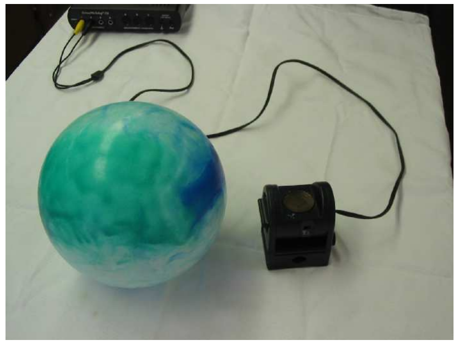

Figure 4.1: The ball and motion detector used in this experiment.

will notice a clicking sound from the motion detector.) Be sure to move your hands out of the way after you release it. A toss of 0.3 to 0.8 m above the motion detector works well. Your teaching assistant can verify your data is correct if you have questions.

- Examine the distance vs. time graph. After the ball leaves your hands, the fall should be in free fall and under the influence of a constant acceleration. The three plots should show a time region with an inverted parabola for the position vs. time plot, a line with a negative slope for the velocity vs time plot, and a flat line (constant) for the acceleration vs. time plot. Identify this time region.

- Use the 'Highlight Region' ( ) icon to outline a box around the data of the time region of interest for each of the three plots. The data points of interest will be shown in yellow. The left most icon on the graph menu is 'scale to fit' ( ). Clicking this icon will display the region of interest outlined with the box.

- Examine each of the three plots in the region of interest. Identify the point in time where the ball left your hands. What is the velocity and acceleration when the ball left your hands? Identify the point where the ball reached its maximum altitude. What is the velocity of the ball at the maximum altitude? What is the acceleration of the ball at the maximum altitude? (Using the annotate icon ( ), you can place a label on different points of the graph.)

- Click on the position vs time plot. Using the fit function button, select a quadratic fit and fit the position vs time plot with a parabola. Note the fit parameters. The quadratic term ('A') is $\frac{1}{2}$g. Why? Calculate the percentage error between the standard value for g (9.8 m/s^2) and the value for g from your fit of the distance vs time graph.

- Click on the velocity vs time plot. Using the fit function, fit the velocity vs time plot with a linear fit. The slope of this line (m) is equal to the g. Why? Calculate the percentage error between the standard value for g (9.8 m/s^2) and the value for g from your fit of the velocity vs time graph.

- Click on the acceleration vs time plot. Click the 'statistics' button ( ) to find the mean value of the acceleration vs time plot. Calculate the percentage error between

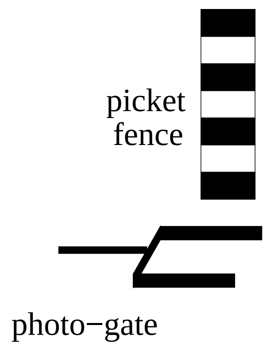

Figure 4.2: Photo-gate and picket fence setup

the standard value for g and this mean value for g.

- Save your data as a '.cap' file using the 'save file as' feature on the file menu. Use a file name you can easily find later if you need to re-examine the data.
- Save each of the three plots with the fits for your lab report.

4.3.3 Measuring g - Picket Fence

This part of the experiment uses a precise timer in the computer and a photo-gate. The photo-gate contains an infrared light source that illuminates a detector on the other side of the photo-gate. The photo-gate detector can determine whenever this infrared beam is blocked. A piece of clear plastic with evenly spaced black bars will be dropped through the photo-gate. The plastic piece is called a picket fence. As the picket fence passes through the photo-gate, the software will measure the time from the leading edge of one bar blocking the beam until the leading edge of the next bar blocks the beam. See Figure 4.2. This timing continues as all bars have passed through the photo-gate. From these measured times and the known distances between the bars (5 cm), the position and velocity for the motion can be calculated and plotted. Values of g to within 1% of the standard value can be obtained using this method.

4.3.4 Procedure

- Open the file 'picket_fence'. (The motion detector is not needed for this part of the experiment so click 'no' when ask if you would like to add the sensor.) Two graphs will appear on the screen. The left graph displays distance vs. time, and the right graph displays velocity vs. time.
- The entire length of the picket fence must be able to fall freely through the photo-gate. To prevent damage to the picket gence, make sure the catch box is directly below the photo-gate.
- Click the 'record' button to start collecting data with the photo-gate. Hold the top of the picket fence and drop it through the photo-gate, releasing it from your grasp

completely before it enters the photo-gate. Be careful when releasing the picket fence. The picket fence must not touch the sides of the photo-gate as it falls and it needs to remain vertical. Click the stop button to end data collection.

- Examine your graphs. Fit the velocity vs time graph with a linear fit. The slope of this line is the acceleration of gravity, g. Why? Fit the position vs time graph with a quadratic fit. Record both values of g in a data table.
- Save your data as a '.cap' file using the 'save file as' feature on the file menu. Use a file name you can easily find later if you need to re-examine the data.
- Repeat the measurement 4 more times for a total of five measurements.
- Calculate the mean and standard deviation (σ) for the five measurements. Compare (percentage error) the mean value with the standard value for g.

4.3.5 Questions to be Addressed in Your Lab Report

1. If an object is moving with constant acceleration, what is the shape of its position vs. time graph? Of the velocity vs. time graph?
2. Does the initial speed of an object have anything to do with its acceleration? Does the direction of an object's initial velocity have anything to do with its acceleration?
3. For the picket fence, list two reasons why your value of g differs from the accepted value. Explain if your reasons would result in a larger or smaller value of g.