REVIEW AND SYNTHESIS: CHAPTERS 13–15

Review Exercises

1. **Strategy** Assume no heat is lost to the air. The potential energy of the water is converted into heating of the water. The internal energy of the water increases by an amount equal to the initial potential energy.

   **Solution** Find the change in internal energy.
   \[ \Delta U = mgh = (1.00 \text{ m}^3)(1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(11.0 \text{ m}) = 108 \text{ kJ} \]

2. **Strategy** Use Eq. (13-22). Form a proportion.

   **Solution** Find the temperature of the nitrogen gas.
   \[ v_{rms} = \sqrt{\frac{3kT}{m}}, \text{ so } \frac{v_N}{v_{He}} = 1 = \frac{m_{He}T_N}{m_NT_{He}} \]
   Therefore, \( T_N = \frac{m_N}{m_{He}}T_{He} = \frac{2 \times 14.00674}{4.00260} \cdot (273.15 \text{ K} + 20.0 \text{ K}) = 2052 \text{ K} = 1779^\circ \text{C}. \]

3. **Strategy** Set the sum of the heat flows equal to zero. Use Eqs. (14-4) and (14-9).

   **Solution**
   
   (a) Find the mass of ice required.
   \[ 0 = Q_w + Q_{ice} = m_wc_w(T_f - T) + m_{ice}c_{ice}(T_f - T_{ice}) = m_wc_w(T_f - T) + m_{ice}(L_f + c_{ice}T_{ice}) \]
   \[ m_{ice} = \frac{m_wc_w(T_f - T)}{L_f + c_{ice}T_{ice}} = \frac{(0.250 \text{ kg})(4.186 \text{ kJ/(kg} \cdot \text{K}))(-25.0 \text{ K})}{333.7 \text{ kJ/kg} + [2.1 \text{ kJ/(kg} \cdot \text{K})](10.0 \text{ K})} = 74 \text{ g} \]

   (b) Find the final temperature of the water, \( T \), which includes the melted ice.
   \[ 0 = Q_w + Q_{ice} \]
   \[ 0 = m_wc_w(T_f - T_{ice}) + m_{ice}c_{ice}(T_f - T_{ice}) + m_{ice}c_w(T_f - T) \]
   \[ 0 = (m_w + m_{ice})c_w(T_f - T_{ice}) + m_{ice}(L_f + c_{ice}T_{ice}) - c_w\left[ m_wT_f + m_{ice}(273.15 \text{ K}) \right] \]
   \[ T = \frac{c_w\left[ m_wT_f + m_{ice}(273.15 \text{ K}) \right] - m_{ice}(L_f + c_{ice}T_{ice})}{(m_w + m_{ice})c_w} \]
   \[ [4.186 \text{ kJ/(kg} \cdot \text{K})][0.250 \text{ kg}](273.15 \text{ K} + 25.0 \text{ K}) + (0.037 \text{ kg})(273.15 \text{ K})] \]
   \[ T = \frac{-(0.037 \text{ kg})(333.7 \text{ kJ/kg} + [2.1 \text{ kJ/(kg} \cdot \text{K})](10.0 \text{ K})}{(0.250 \text{ kg} + 0.037 \text{ kg})(4.186 \text{ kJ/(kg} \cdot \text{K})}] = -273.15 \text{ K} = 11^\circ \text{C} \]

4. **Strategy** Determine how much larger the volume of the Pyrex container is than the volume of the water after the temperature decrease. Use Eq. (13-7).

   **Solution** Find the amount of water that can be added.
   \[ \frac{\Delta V}{V_0} = \beta \Delta T, \text{ so } \Delta V_{Pyrex} - \Delta V_{water} = V_0(\beta_{Pyrex} - \beta_{water})\Delta T \]
   \[ = (40.0 \text{ L})(9.75 \times 10^{-6} \text{ K}^{-1} - 207 \times 10^{-6} \text{ K}^{-1})(20.0^\circ \text{C} - 90.0^\circ \text{C}) = 0.552 \text{ L}. \]
5. **Strategy** Use the ideal gas law.

**Solution** Find the number of moles of air when the balloon is at 40.0°C.

\[ PV = nRT, \text{ so } n_2 = \frac{PV}{RT_2} = \frac{(1.00 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})(12.0 \text{ m}^3)}{[8.314 \text{ J/(mol·K)}](273.15 \text{ K} + 40.0 \text{ K})} = 467 \text{ mol}. \]

6. (a) **Strategy** The maximum power emission is inversely proportional to the absolute temperature. Use Wien’s law.

**Solution** Compute the surface temperature of the star.

\[ T = \frac{2.898 \times 10^{-3} \text{ m·K}}{\lambda_{\text{max}}} = \frac{2.898 \times 10^{-3} \text{ m·K}}{700.0 \times 10^{-9} \text{ m}} = 4140 \text{ K} \]

(b) **Strategy** Use Stefan’s law of blackbody radiation.

**Solution** Compute the power radiated.

\[ 3 = \sigma A T^4 = [5.670 \times 10^{-8} \text{ W/(m}^2 \cdot \text{K}^4)](4\pi(7.20 \times 10^8 \text{ m})^2)(4140 \text{ K})^4 = 1.09 \times 10^{26} \text{ W} \]

(c) **Strategy** Intensity is power radiated per unit area.

**Solution** Compute the intensity measured by the Earth-based observer.

\[ I = \frac{3}{A} = \frac{1.085 \times 10^{26} \text{ W}}{4\pi(9.78 \text{ ly})^2(9.461 \times 10^{15} \text{ m/ly})^2} = 1.01 \times 10^{-9} \text{ W/m}^2 \]

7. (a) **Strategy** Use Eqs. (14-12) and (14-13).

**Solution** Find the thermal resistance for each material.

\[ R_{\text{wood}} = R_w = \frac{d_w}{\kappa_w A} \text{ and } R_{\text{insulation}} = R_i = \frac{d_i}{\kappa_i A}. \]

Compute the rate of heat flow.

\[ 3 = \frac{\Delta T}{R_w + R_i} = \frac{A\Delta T}{d_w + d_i} \cdot \frac{0.0100 \text{ m}}{0.13 \text{ W}/(\text{m·K})} + \frac{0.0300 \text{ m}}{0.038 \text{ W}/(\text{m·K})} = 320 \text{ W} \]

(b) **Strategy** The area of the insulated wall has been reduced by half, so the rate of heat flow through it is reduced by half. The other half is glass. The total rate of heat flow is the sum of the two rates.

**Solution** Find the rate of heat flow.

\[ 3 = \frac{\Delta T}{R_g} = \frac{A\Delta T}{d_g} \cdot \frac{\kappa_g}{\kappa_g} \text{, so} \]

\[ 3_{\text{total}} = \frac{\kappa_g A\Delta T}{d_g} + \frac{3}{2} = \frac{[0.63 \text{ W/(m·K)}](1/2)(2.74 \text{ m})(3.66 \text{ m})(23.0°C - (-5.00°C)]}{0.00500 \text{ m}} + \frac{324 \text{ W}}{2} = 18 \text{ kW} \]
8. (a) **Strategy** The net work done in one cycle is equal to the area inside the graph.

**Solution** Compute the net work done per cycle.

\[ W = (4.00 \text{ atm} - 1.00 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})(0.800 \text{ m}^3 - 0.200 \text{ m}^3) = 182 \text{ kJ} \]

(b) **Strategy and Solution** The net heat flow into the engine is equal to the work done per cycle, so

\[ Q_{\text{net}} = 182 \text{ kJ} \]

9. (a) **Strategy** Use the ideal gas law.

**Solution** Find the pressure at point A, which is the same as the pressure at point D.

\[ P_D = P_A = \frac{nRT_A}{V_A} = \frac{(2.00 \text{ mol})(8.314 \text{ J/(mol·K)})(800.0 \text{ K})}{1.50 \text{ m}^3} = 8.87 \text{ kPa} \]

Find the temperature at point D.

\[ T_D = \frac{P_D V_D}{nR} = \frac{(8.87 \times 10^3 \text{ Pa})(2.25 \text{ m}^3)}{(2.00 \text{ mol})(8.314 \text{ J/(mol·K)})} = 1200 \text{ K} \]

(b) **Strategy** The net work done on the gas is equal to the area inside the graph.

**Solution** Find the net work done on the gas as it is taken through four cycles.

\[ W = 4(8.87 \text{ kPa} - 1.30 \text{ kPa})(2.25 \text{ m}^3 - 1.50 \text{ m}^3) = 23 \text{ kJ} \]

(c) **Strategy** The internal energy of an ideal monatomic gas is given by \( U = \frac{3}{2} nRT \).

**Solution** Compute the internal energy of the gas at point A.

\[ U = \frac{3}{2} nRT = \frac{3}{2}(2.00 \text{ mol})(8.314 \text{ J/(mol·K)})(800.0 \text{ K}) = 20.0 \text{ kJ} \]

(d) **Strategy and Solution** The total change in internal energy in four complete cycles is \( 0 \), since the change in temperature is zero.

10. **Strategy** The temperature is constant and the heat entering the system is \( Q = mL_f \).

**Solution** Find the change in entropy of the ice.

\[ \Delta S = \frac{Q}{T} = \frac{mL_f}{T} = \frac{(2.00 \text{ kg})(333.7 \text{ kJ/kg})}{273.15 \text{ K} + 0.0 \text{ K}} = 2.44 \text{ kJ/K} \]
11. **Strategy** The gravitational potential energy of the steel ball is converted into heat. Set the sum of the heat flows equal to zero.

**Solution** Find the final temperature of the system, \( T \).

\[
0 = U_s + Q_s + Q_w \\
0 = -m_s c_s \Delta T_s + m_w c_w \Delta T_w \\
0 = -m_s g h + m_s c_s (T - T_s) + m_w c_w (T - T_w) \\
0 = m_w c_w T_w + m_s (g h + c_s T_s) \\
T = \frac{m_w c_w T_w + m_s (g h + c_s T_s)}{m_s c_s + m_w c_w}
\]

\[
T = \left( \frac{(4.50 \text{ L})(1 \text{ kg/L})(4186 \text{ J/kg} \cdot \text{K})(10.1^\circ \text{C}) + (7.30 \text{ kg})(9.80 \text{ m/s}^2)(10.0 \text{ m}) + (450 \text{ J/kg} \cdot \text{K})(15.2^\circ \text{C})}{(7.30 \text{ kg})(450 \text{ J/kg} \cdot \text{K}) + (4.50 \text{ L})(1 \text{ kg/L})(4186 \text{ J/kg} \cdot \text{K})} \right) = 10.9^\circ \text{C}
\]

12. (a) **Strategy** Use the definition of pressure.

**Solution** Find the weight of the car.

\[
P = \frac{F}{A} \quad \text{so} \quad F = PA = (36.0 \text{ lb/in}^2)(2 \times 24.0 \text{ in}^2) = 3170 \text{ lb}
\]

(b) **Strategy** Since the contact areas of the front tires are greater than those of the back tires, the front half of the car weighs more than the back half. Use the definition of center of mass.

**Solution** Compute the \( y \)-coordinate of the car’s center of mass. Since \( F = ma = PA \), the mass each tire supports is proportional to the contact area, so the contact areas can be used instead of the masses when computing the center of mass. Due to symmetry, the front tires can be combined, as well as the rear tires.

\[
y_{\text{CM}} = \frac{m_{\text{front}} y_{\text{front}} + m_{\text{rear}} y_{\text{rear}}}{m_{\text{front}} + m_{\text{rear}}} = \frac{A_{\text{front}} y_{\text{front}} + A_{\text{rear}} y_{\text{rear}}}{A_{\text{front}} + A_{\text{rear}}}
\]

\[
y_{\text{CM}} = \frac{(48.0 \text{ in}^2)(0) + (40.0 \text{ in}^2)(-7.00 \text{ ft})}{48.0 \text{ in}^2 + 40.0 \text{ in}^2} = -3.18 \text{ ft}
\]

13. **Strategy** Use Fourier’s law of heat conduction for the copper rod and the heat of fusion for the ice.

**Solution** Find the rate of melting.

\[
\frac{Q}{\Delta t} = \frac{m L_f}{\Delta t} = \kappa A \frac{\Delta T}{d} \quad \text{so} \quad \frac{m A \Delta T}{L_f d} = \frac{[401 \text{ W/(m} \cdot \text{K})](0.0100 \text{ m})^2}{(333.7 \text{ J/g})(1.00 \text{ m})/1 \text{ h}} = 136 \text{ g/h}
\]

14. (a) **Strategy and Solution**

The boiling temperature of water varies with pressure. If the pressure is high, the water molecules are pushed close together, making it harder for them to form a gas. (Gas molecules are farther apart from each other than are liquid molecules.) A higher pressure raises the temperature at which the coolant fluid will boil.

(b) **Strategy and Solution**

If you were to remove the cap on your radiator without first bringing the radiator pressure down to atmospheric pressure, the fluid would suddenly boil, sending out a jet of hot steam that could burn you.
15. Strategy  The work done per stroke (cycle) is equal to the average pressure times the change in volume. The average power output is equal to the operating frequency times the work per stroke.

Solution  Find the operating frequency $f$. Let $p_{av}$ be the average power output.

$$fW_{stroke} = f^2p_{av}\Delta V = p_{av}, \text{ so } f = \frac{p_{av}}{p_{av}\Delta V} = \frac{27.6\times10^3}{(1.3\times10^5 \text{ Pa})(0.150 \text{ m/2})^2(0.200 \text{ m})} = 60 \text{ Hz}$$

16. Strategy and Solution

(a) Since the blocks are made of the same material and are at the same temperature, they will have the same internal energy if they have the same mass.

(b) Since they are at the same temperature, there is no net energy transfer between the two blocks.

(c) The blocks need not touch each other in order to be in thermal contact. They can be in thermal contact due to convection and radiation.

17. (a) Strategy  The maximum possible efficiency is that of a reversible engine.

Solution  Compute the maximum possible efficiency.

$$e_r = 1 - \frac{T_C}{T_H} = 1 - \frac{323 \text{ K}}{535 \text{ K}} = 0.396 \text{ or } 39.6\%$$

(b) Strategy  Use energy conservation and the definition of efficiency of an engine.

Solution  Find the rate at which heat must be removed by means of a cooling tower.

$$\frac{W_{net}}{\Delta t} = \frac{Q_H}{\Delta t} - \frac{Q_C}{\Delta t} = \frac{W_{net}}{\Delta t} - \frac{Q_C}{\Delta t}, \text{ so }$$

$$\frac{Q_C}{\Delta t} = \frac{W_{net}}{\Delta t} - \frac{W_{net}}{\Delta t} \left(1 - \frac{1}{0.500e_r} - 1\right) = (1.23\times10^8 \text{ W}) \left(\frac{1}{0.500(0.396)} - 1\right) = 4.98\times10^8 \text{ W}.$$
18. (a) **Strategy** Use the ideal gas law, \( PV = nRT \). Draw a qualitative diagram.

**Solution** First, the temperature is constant, so \( P \propto V^{-1} \). Since the volume is reduced to one-eighth of its initial size, the pressure increases by a factor of eight. Next, the volume is constant, while the temperature and pressure increases. Then, the temperature is again constant. Finally, the volume is constant as the temperature and pressure decreases. The \( P-V \) diagram is shown.

![P-V diagram](image)

(b) **Strategy** Refer to the diagram in part (a). Calculate the quantities for each step in the cycle. Note that the gas is diatomic.

**Solution** Step 1, isothermal process:
The work done on the gas is 
\[
W = nRT \ln \frac{V_i}{V_f} = (2.00 \text{ mol})[8.314 \text{ J/(mol·K)}](325 \text{ K}) \ln 8 = 11.2 \text{ kJ}.
\]
The change in the internal energy of the gas is 0 for an isothermal process. The heat transferred is \( Q = -W = -11.2 \text{ kJ} \).

Step 2, isochoric process:
Without a displacement, work cannot be done, so \( W = 0 \).
The change in the internal energy of the gas is equal to the heat that enters the system, so the change in internal energy and the heat transferred are
\[
\Delta U = Q = nC_v \Delta T = n \left( \frac{5}{2} \right) R \Delta T = \frac{5}{2} (2.00 \text{ mol})[8.314 \text{ J/(mol·K)}](985 \text{ K} - 325 \text{ K}) = 27.4 \text{ kJ}.
\]

Step 3, isothermal process:
The work done on the gas is 
\[
W = nRT \ln \frac{V_i}{V_f} = (2.00 \text{ mol})[8.314 \text{ J/(mol·K)}](985 \text{ K}) \ln \frac{1}{8} = -34.1 \text{ kJ}.
\]
The change in the internal energy of the gas is 0 for an isothermal process. The heat transferred is \( Q = -W = 34.1 \text{ kJ} \).

Step 4, isochoric process:
Without a displacement, work cannot be done, so \( W = 0 \).
The change in the internal energy of the gas is equal to the heat that enters the system, so the change in internal energy and the heat transferred are
\[
\Delta U = Q = nC_v \Delta T = n \left( \frac{5}{2} \right) R \Delta T = \frac{5}{2} (2.00 \text{ mol})[8.314 \text{ J/(mol·K)}](325 \text{ K} - 985 \text{ K}) = -27.4 \text{ kJ}.
\]
The results of the processes and the totals are shown in the table. (Note that the totals for work and heat differ slightly from the sums of the values for each step due to round-off error.)

<table>
<thead>
<tr>
<th>Process</th>
<th>( W ) (kJ)</th>
<th>( \Delta U ) (kJ)</th>
<th>( Q ) (kJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>11.2</td>
<td>0</td>
<td>-11.2</td>
</tr>
<tr>
<td>Step 2</td>
<td>0</td>
<td>27.4</td>
<td>27.4</td>
</tr>
<tr>
<td>Step 3</td>
<td>-34.1</td>
<td>0</td>
<td>34.1</td>
</tr>
<tr>
<td>Step 4</td>
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<td>-27.4</td>
</tr>
<tr>
<td>Total</td>
<td>-22.8</td>
<td>0</td>
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</table>

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**Review and Synthesis: Chapters 13–15**

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College Physics
(c) **Strategy** The efficiency is equal to the ratio of the net work done by the gas to the heat transferred into the gas.

**Solution** The work done by the gas is negative the work done on the gas.

\[ W_{\text{net}} = (-22.8 \text{ kJ}) = 22.8 \text{ kJ} \] 

and the heat transferred into the gas is \( Q_{\text{in}} = 27.4 \text{ kJ} + 34.1 \text{ kJ} = 61.5 \text{ kJ} \).

The efficiency of the engine is \( e = \frac{W_{\text{net}}}{Q_{\text{in}}} = \frac{22.8 \text{ kJ}}{61.5 \text{ kJ}} = 0.371 \) or 37.1%.

(d) **Strategy** Use Eq. (15-17).

**Solution** Compute the efficiency of a Carnot engine operating at the same extreme temperatures.

\[ e_r = 1 - \frac{T_C}{T_H} = 1 - \frac{325 \text{ K}}{985 \text{ K}} = 0.670 \text{ or 67.0%} \]

19. **Strategy** Use the ideal gas law and Hooke’s law.

**Solution** Find the final pressure in terms of the initial pressure.

\[ \frac{P_2V_2}{P_1V_1} = \frac{nRT}{nRT} = 1, \text{ so } P_2 = \frac{V_1}{V_2}P_1 = \frac{\frac{1}{2}\pi d^2h}{\frac{1}{2}\pi d^2(h + \Delta x)}P_1 = \frac{h}{h + \Delta x}P_1. \]

The magnitudes of the forces due to the spring, on the outside of the piston, and on the inside of the piston are \( k\Delta x \), \( k\Delta x + P_{\text{atm}}A \), and \( P_2A \), respectively. Set the forces equal, substitute for \( P_2 \), and solve for \( \Delta x \).

\[ k\Delta x + P_{\text{atm}}A = P_2A = \frac{h}{h + \Delta x}P_1A, \text{ so } k(\Delta x)^2 + (kh + P_{\text{atm}}A)\Delta x - Ah(P_1 - P_{\text{atm}}) = 0. \]

Thus, \( \Delta x = \frac{-(kh + P_{\text{atm}}A) \pm \sqrt{(kh + P_{\text{atm}}A)^2 + 4kAh(P_1 - P_{\text{atm}})}}{2k} \).

Substituting \( k = 1.00 \times 10^3 \text{ N/m} \), \( h = 0.100 \text{ m} \), \( P_{\text{atm}} = 1.013 \times 10^5 \text{ Pa} \), \( A = \frac{1}{4}\pi d^2 = \frac{1}{4}\pi (0.0500 \text{ m})^2 \), and \( P_1 = 5.00 \times 10^5 \text{ Pa} \), we find that \( \Delta x = 0.168 \text{ m} \), where \( -0.467 \text{ m} \) is extraneous, since the gas expands.

**MCAT Review**

1. **Strategy and Solution** According to the second law of thermodynamics, heat never flows spontaneously from a colder body to a hotter body, therefore, heat will not flow from bar A to bar B. The correct answer is [C].

2. **Strategy** Assume that the specific heat capacity of seawater is approximately the same at 0°C and 5°C.

**Solution** Find the approximate temperature \( T \).

\[ 0 = Q_0 + Q_2 = mc(T - 0°C) + mc(T - 5°C), \text{ so } 2T = 5°C \text{ or } T = 2.50°C. \]

The correct answer is [B].

3. **Strategy** Use the latent heat of fusion for water.

**Solution** The heat gained by the ice when melting is \( Q = mL_f = (0.0180 \text{ kg})(333.7 \text{ kJ/kg}) = 6.01 \text{ kJ} \).

The correct answer is [C].
4. **Strategy and Solution** Since \( e = 1 - Q_C/Q_H = 1 - T_C/T_H \), decreasing the exhaust temperature will increase the steam engine’s efficiency. The correct answer is **B**.

5. **Strategy and Solution** Since refrigerators remove heat by transferring it to a liquid that vaporizes, refrigerators are primarily dependent upon the heat of vaporization of the refrigerant liquid. The correct answer is **A**.

6. **Strategy and Solution** Steam is generally at a higher temperature than water and the specific heat of steam is lower than that of water, so water would be more effective than steam for changing steam to water. Circulating water brings more mass of water in contact with the condenser than stationary water, so it can carry away heat at a faster rate, therefore, it would be more effective for changing steam to water. The correct answer is **D**.

7. **Strategy and Solution** Since it is not possible to convert all of the input heat into output work, the amount of useful work that can be generated from a source of heat can only be less than the amount of heat. The correct answer is **A**.

8. **Strategy and Solution** The internal energy of the steam is converted into mechanical energy as it expands and moves the piston of the steam engine to the right, therefore, the correct answer is **C**.

9. **Strategy and Solution** The refrigerant must be able to vaporize (boil) at temperatures lower than the freezing point of water so that it can carry away heat (as a gas) from the contents of the refrigerator (which contain water) to cool and possibly freeze the contents. The correct answer is **B**.

10. **Strategy** The heat transferred to the water by the heaters was \( Q_w = m_w c_w \Delta T_w \). The heat required for the oil is \( Q_o = m_o c_o \Delta T_o \).

**Solution** Form a proportion and use the temperature changes of the oil and water and the specific heat and the specific gravity of the oil to obtain a ratio of heat required for the oil to that transferred to the water.

\[
\frac{Q_o}{Q_w} = \frac{m_o c_o \Delta T_o}{m_w c_w \Delta T_w} = \frac{(0.7 m_w)(0.60 c_w)(60 - 20)}{m_w c_w (100 - 20)} = 0.21
\]

So, 21% of the amount of heat transferred to the water is required to heat the oil to 60°C. Assuming the heaters work at the same rate for both the water and the oil, the time required to raise the temperature of the oil from 20°C to 60°C is 0.21(15 h) = 3.2 h. The correct answer is **A**.

11. **Strategy and Solution** The high pressure would increase the pressure on the plug, making it more difficult to lift. The pressure difference between the air in the tank and the air outside of the tank would increase the fluid velocity when the tank is drained, thus, decreasing the time required to drain the tank. The time required to heat the oil would be the least likely affected, since the oil is fairly incompressible. The correct answer is **A**.