(1) A particle of mass m is confined in a one-dimensional box of length a:

$$V(x) = \begin{cases} \infty, & \text{if } x < 0; \\ 0, & \text{if } 0 \le x \le a; \\ \infty, & \text{if } x > a. \end{cases}$$

At $t = 0$ the particle is in a state with normalized wave function

$$\psi(x, t = 0) = \sqrt{\frac{8}{5a}} \sin(\frac{\pi x}{a}) [1 + \cos(\frac{\pi x}{a})]$$

1. Sketch $\psi(x, 0)$.

2. What are the possible results of an energy measurement at $t = 0$? What is the average energy of the system at $t = 0$?

3. What is the wave function $\psi(x, t)$ at a later time t?

4. What is the average energy of the system at a later time t?

5. At time t, what is the probability to find the particle in the left half of the box (i.e., $0 < x < a/2$)? Argue that your result makes qualitative sense at $t = 0$ based on the sketch in (1).

6. At time t, what is the current (probability flux) $j(x, t)$ at $x = a/2$? Relate it to your answer in part (5).