(2) (*Adopted from Gottfried and Sakurai*) Let X be a Hermitian operator and $\{|E_n\rangle\}$ the eigenstates of the Hamiltonian H of the system. Let $X^{(H)}$ denote the operator X in the Heisenberg picture.

1. Show that
$$\langle E_0|[\frac{dX^{(H)}}{dt}, X^{(H)}]|E_0\rangle = \frac{2}{i\hbar}\sum_n (E_n - E_0)\, |\langle E_0|X|E_n\rangle|^2.$$

2. Let x be the coordinate of particle (of mass m) in one dimension in a static potential, i.e., with a Hamiltonian
$$H = \frac{p^2}{2m} + V(x).$$
Use (1) to show that
$$\sum_n (E_n - E_0)\, |\langle E_0|x|E_n\rangle|^2 = \frac{\hbar^2}{2m}.$$
This is known as the *dipole sum rule*. It played an important role in the discovery of quantum mechanics, and is useful in a variety of problems (such as a charged particle moving through bulk materials).