(3) Consider two identical particles in a one-dimensional Harmonic potential. We will work in reduced units: $\hbar = m = 1$, where m is the particle mass. The two particles interact with a potential: $V_{\text{int}} = \alpha x_1 x_2$, where x_1 are x_2 are the coordinates of the two particles, and α is a parameter which satisfies $|\alpha| < 1$.

- What is the ground-state energy of the system if the particles have spin 0?

- What is the ground-state energy of the system if the particles have spin $1/2$ and they are both in the $|s_z = \uparrow\rangle$ state?

- What is the ground-state energy of the system if the particles have spin $1/2$ but are in a spin singlet?

Note: This problem can be solved exactly.