To appear in the 13th AIP Proc. RF Power in Plasma

RF heating in a tokamak cavity[1]

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Abstract. We consider the problem of computing the global response of a tokamak to RF-antenna driving, including conversion at the ion-hybrid resonance layer. The tokamak is modeled as a 2-D circular cavity (a poloidal cross-section). An antenna launches a family of magnetosonic (MS) rays. The amplitude and phase of the MS wave field are transported using eikonal techniques: the phase is transported using the standard phase integral, while the amplitude is calculated using the van Vleck formula which accounts for the convergence or divergence of neighboring rays. As each ray of this family crosses the ion-hybrid (IH) resonance it is partially transmitted, partially reflected and partially converted into an IH wave which remains confined to the resonance layer. This MS/IH conversion process is described by an S-matrix. The S-matrix can be evaluated using previously developed techniques. The transmitted and reflected MS rays now propagate from the resonance layer. They are globally confined and are reflected at the edge of the plasma. Hence, they will re-enter the resonance layer. At each resonance crossing new families of rays are created and some fraction of the MS wave energy and action is converted into the IH wave, eventually damping on the background plasma. The resulting field distribution in the cavity will be a superposition of this multitude of ray families. Fine-scale structure is observed to emerge due to caustic formation. We focus our attention on this iterated conversion to the IH wave and ask how the energy leakage affects the overall cavity response and the spatial distribution of energy absorbed as a function of frequency.

INTRODUCTION

In this short note we summarize the logic of a new calculational approach to the study of RF heating in tokamaks. We provide only a qualitative summary of the application to a very simplified model. Details will be presented elsewhere.

We start with a cold DT plasma ($n_D = n_T = n_e/2$) in tokamak geometry and consider a poloidal cross section in the (x, z)-plane (see lower part of Fig. 1). All fields are assumed to have time variation $e^{-i\omega t}$. The magnetic field strength is assumed to vary as $B(x) \approx B_0(1 - \frac{x}{L_B})$. Projecting the 3×3 cold plasma dispersion

[1]) This research was supported by the US Department of Energy under contracts DE-AC03-76SF00098 and DE-FG02-96ER54344.

tensor onto the *uncoupled* polarizations $\hat{\mathbf{e}}_M \equiv \hat{x} + i\hat{y}/\sqrt{2}$, $\hat{\mathbf{e}}_H \equiv \hat{x}$ we arrive at the canonical 2×2 form [1]

$$\begin{pmatrix} D_H & \eta \\ \eta & D_M \end{pmatrix} \tag{1}$$

with $D_H \equiv x - x_H(\omega)$, $D_M \equiv \frac{\omega^2}{c_A^2} - k_\perp^2$ and η a constant (we assume that $k_\| = 0$ and that $\mathbf{B}$ is purely toroidal). For the applications of present interest the coupling constant $\eta \sim \mathcal{O}(1)$ giving weak transmission ($\tau = e^{-\pi\eta^2} \approx 4\%$).

A family of MS rays is launched by an antenna (Fig. 1). The peregrinations of this family and its descendants are followed as they propagate through the system. We focus particular attention on the return maps as the families repeatedly cross the resonance layer. The use of *families* of rays (*lagrange manifolds*) allows us to calculate the amplitude and phase transported along the rays. Following *single* rays is insufficient in this regard [2]. The use of semi-classical techniques to calculate cavity eigenmodes is a well-established area (see, e.g. [3,4] for overviews, [5] for an exposition of 'wave maps', and [7] for an example of 'ray splitting').

The new aspect of the present study is the presence of the hybrid resonance. Even without damping, this resonance leads to energy leakage from the cavity and results in a finite Q. Hence, we are *not* calculating eigenmodes – which do not exist – but the *cavity response*. There is some formal similarity with the work of [6] concerning the semi-classical analysis of semi-conductor nanostructures. In that case, however, the cavity is *physically open*, while in the present case the magnetosonic rays are spatially confined while the hybrid rays can escape *by propagating in k-space*.

The resonance crossing is analyzed as a two-step process using the modular approach of Ye and Kaufman [8] (see caption Fig. 1). Referring now to Fig. 2: the incoming rays from the antenna (ψ_0) enter the hybrid resonance at 1. The field ϕ_H which exits the upper conversion at the position labeled 7 is the hybrid mode excited by the antenna. The 'cavity response' is the linear operator $\hat{C}$ defined as: $\phi_H = \hat{C}(\omega)\psi_0$.

In Fig. 3 we show the results of a typical numerical study. The initial family of 1000 rays was focused at the magnetic axis with a gaussian half-width consistent with the antenna geometry of DIII-D (i.e. symmetric about the axis with maximum angular extension of $\pm 26°$ [10]). Because the transmission coefficient is so small ($\tau \sim 4\%$), only ray paths which involve a single transmission are retained, preventing exponential blow-up in the number of rays. The frequency is $.525\omega_{H0}$ and $k_\perp^2 = 400 m^{-2}$ [11]. One hundred crossings are used to construct the field. By this stage, all but $\approx 1\%$ of the initial energy has leaked out of the cavity. The entire calculation takes less than one minute on an SGI-O2 workstation.

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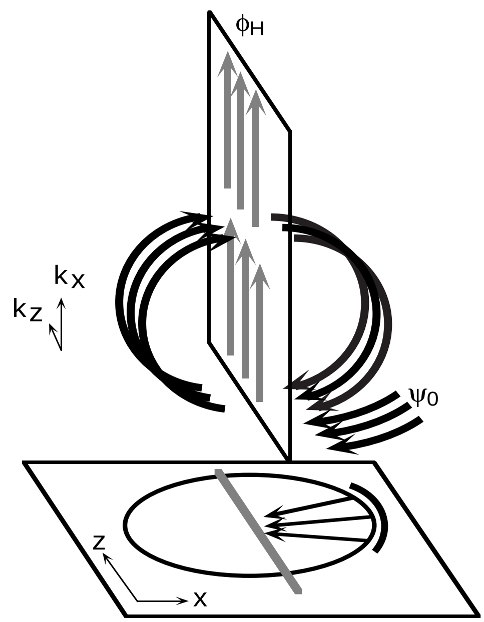

FIGURE 1. Phase space diagram of the resonance crossing. A group of rays forming the incoming family ψ_0 (a lagrange manifold) is launched by the antenna and crosses the hybrid resonance at x_H. This family is the lower set of bold black rays on the right. These are partially transmitted to the high field side of x_H and partially converted. The converted rays propagate upward in k-space until they once again satisfy the MS dispersion relation, where they partially convert again to form the reflected MS rays. The remainder of these hybrid rays are transmitted and contribute to the outgoing hybrid disturbance, ϕ_H. Meanwhile, the magnetosonic rays which were transmitted at the first resonance have reflected off the inner boundary of the plasma and can once again cross the resonance. Here they can be transmitted – and superpose with the reflected rays already present – or they can convert to the hybrid mode. The MS rays now leaving the resonance region moving to the right reflect at the plasma boundary and re-enter the resonance. This process repeats itself *ad infinitum*.

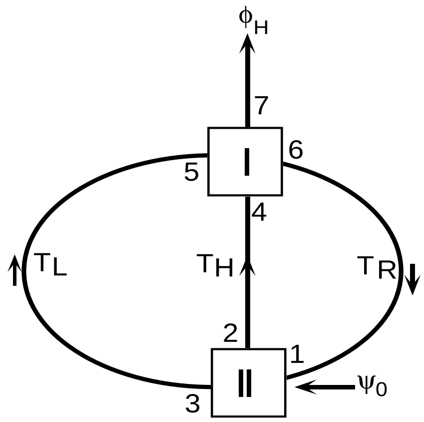

FIGURE 2. Diagram of the possible paths a disturbance may take as it propagates from the antenna input, ψ_0, to the output hybrid channel, ϕ_H. The resulting steady state field set up by the antenna is a linear superposition of all possible paths through this flow diagram.

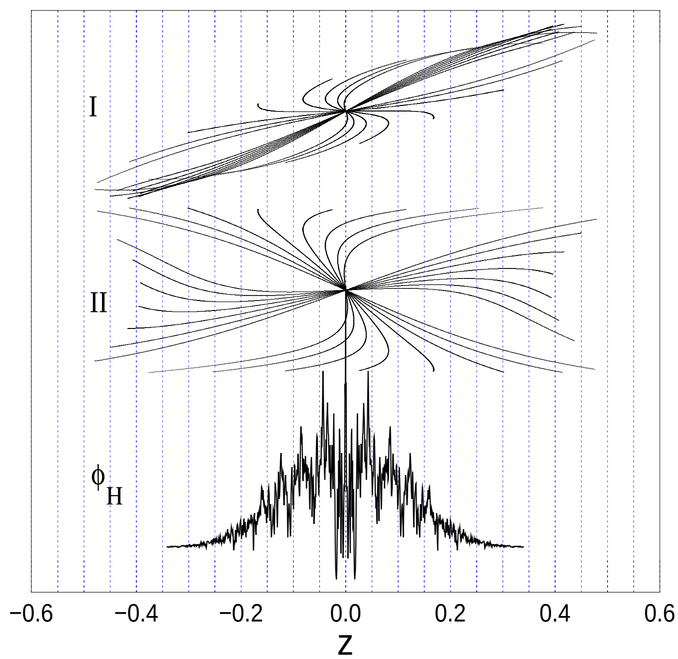

FIGURE 3. Poincare surfaces of section in the $z-k_z$ plane for rays crossing the hybrid resonance at the upper crossing (I) and the lower crossing (II). The hybrid field excitation, $\phi_H(z)$, is shown just above the upper crossing (labeled as position 7 in Fig. 2). Only 15 crossings were used to illustrate the poincare surfaces of section, while 100 were used to generate the hybrid field. The vertical scales are arbitrary and the poincare surfaces of section have been shifted for ease of viewing. The dashed vertical lines are meant to guide the eye to the relationship between fine scale structure in the field and the corresponding caustics (folds of the lagrange manifolds). (Notice that not all peaks in the field have an associated fold due to the fact that not all 100 lagrange manifolds have been shown, to avoid clutter.)