Electric Field Structure in 2-dimensional Iterated Mode Conversion

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Expanded Abstract and Introduction

In the poloidal plane (x,y) of a tokamak, we study the iterated conversion of an injected magnetosonic (M) wave to an ion-hybrid (H) wave, for given frequency and toroidal mode number. Each of the two wave-types is represented by a set of ray manifolds; these are generated by conversion, whenever a manifold of one type intersects the dispersion surface of the other type. The ray manifolds act as a skeleton for the wave fields. We present numerical calculations of the set of manifolds, and discuss how to calculate the wave field, **E**(x,y), explicitly.

Outline

1. Motivation
2. Review of ray tracing in (x,y,k_x,k_y) phase space, including iterated conversion
3. Algorithm for construction of the wave field
4. Conclusions

Motivation

We provide a ray-based alternative to the full wave approach for the analysis of linear wave conversion in tokamak geometry. Geometric optics is computationally much more efficient, since here we solve ODEs (the ray equations) rather than PDEs. This new approach can easily deal with the fine spatial scale structure, something which is difficult for full wave methods. Having successfully studied the 1d problem, we now turn to the case of 2 spatial dimensions, which introduces many new features, such as the distinction between 1d rays, 2d ray manifolds, and 3d dispersion surfaces in 4d phase space. Here we idealize the geometry, plasma properties, and antenna effects, in order to concentrate on the essential consequences of the new features. In future work we plan to introduce more realism into our model.

Idealized physical model

- Cold D-T plasma ($n_D=n_T=n_e/2$), uniform density
- $\mathbf{E}(x,y,z;t) = e^{i(mz-\omega t)}\mathbf{E}(x,y)$ $\quad [E_z = 0]$
- $\mathbf{B}_0(x) = B_0(1 - x/L_B)\hat{z}$
- $\mathbf{D}(x,y;-i\partial_x,-i\partial_y;\omega)\bullet\mathbf{E}(x,y)=0$
  2x2 cold plasma dispersion tensor, ignoring $k_{\|} = m$ (for the time being).
- Project onto 2 *uncoupled* polarizations:
  M=Magnetosonic, H=ion-hybrid

$$\hat{e}_M \equiv \frac{\hat{x}+i\hat{y}}{\sqrt{2}} \; (right\ circular); \qquad \hat{e}_H \equiv \hat{k} \; (longitudinal)$$

$$\mathbf{E}(x,y) = E_M(x,y)\hat{e}_M - \nabla\phi_H(x,y)$$

$$\begin{pmatrix} \hat{D}_M & \eta \\ \eta & \hat{D}_H \end{pmatrix} \begin{pmatrix} E_M \\ E_H \end{pmatrix} = 0$$

$$E_H = -ik\phi_H$$

$$D_M = R - \tfrac{1}{2} N^2 = \left(\frac{\omega^2}{c_A^2} - k^2\right)\kappa_M \; ;$$

$$D_H = \varepsilon_\perp = (x - x_H(\omega))\kappa_H$$

$$\eta = R / \sqrt{2}$$

The tokamak as a resonant cavity

- Launch a family of rays from the antenna. Magnetosonic (M) wave field injected is $E_{M0}(x,y)$.
- Calculate phase and amplitude fields, using standard phase integral and van Vleck determinant.
- Use modular approach: local conversion processes connected by propagation.

Algorithm for construction of the wave fields

1. The incident M field before the first conversion
2. The first conversion process
3. The transmitted M and converted H field after first conversion
4. The H field between the 1st and 2nd conversions
5. Reflection of the transmitted M from the inner edge of the plasma
6. The 2nd conversion: interference effects from M and H
7. Fields after 2nd conversion
8. Iteration

1. The incident M field before the first conversion

Free propagation without caustics:

$$E_M(x,y) = \tilde{E}_M(x,y) \exp i\theta_M(x,y)$$

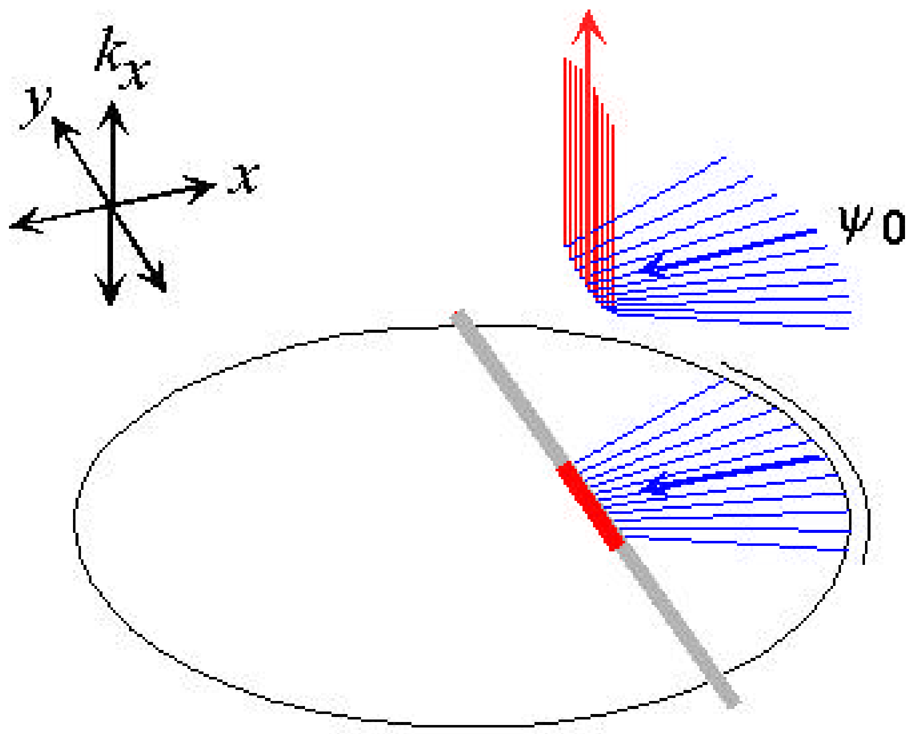
$$\theta_M(\mathbf{x}) = \int_{\mathbf{x_0}}^{\mathbf{x}} \mathbf{k}(\mathbf{x}') \cdot \mathrm{d}\mathbf{x}'$$

(Integration is along a ray.)

$$\tilde{E}_M(\mathbf{x}) = \left| det \frac{\partial \mathbf{x}}{\partial \mathbf{x}_0} \right|^{-1/2} \tilde{E}_M(\mathbf{x}_0)$$

(Alternatively: can express the local variation of amplitude in terms of ray divergence.)

2. The first conversion process



(The transmitted M rays are not shown, for clarity.)

- At the conversion of each ray, use the S-matrix connection formula:

$$\begin{pmatrix} E_M^{(out)} \\ E_H^{(out)} \end{pmatrix}' = \begin{pmatrix} \tau & \beta \\ -\beta^* & \tau \end{pmatrix} \begin{pmatrix} E_M^{(in)} \\ E_H^{(in)} \end{pmatrix}'$$

$$\tau = exp(-\pi\bar{\eta}^2)$$

$$\bar{\eta} = \eta / \sqrt{B}; \quad B = |\{ D_M, D_H \}|$$

The Poisson bracket is evaluated at the conversion

$$\beta = \frac{\sqrt{2\pi\tau}}{\bar{\eta}\Gamma(-i\bar{\eta}^2)}$$

See: Tracy & Kaufman, PRE **48** (1993) 2196.

Matching conditions

Including the coupling η, the eikonal form for the M field is modified to become:

$$E_M(x,y)=\tilde{E}_M(x,y)[\exp i\theta_M(x,y)]\left(\frac{x-x_H}{x_0-x_H}\right)^{i\bar{\eta}^2}$$

- Now evaluate the amplitude and eikonal phase at the conversion point (x_c,y_c). This point is the intersection of the uncoupled M ray with the hybrid dispersion surface, $D_H=0$, i.e., $x=x_H$.
- Expand the eikonal phase θ_M to quadratic order in $(\mathbf{x}-\mathbf{x}_c)$.
- We have developed an algebraic algorithm (to be published) for constructing the eikonal phase θ_H for the emerging hybrid wave, from θ_M.

Matching conditions (contd.)

•Connect the outgoing H amplitude to the incoming M amplitude, using the S-matrix.

$$\tilde{E}_H(-k_0, y_c)(-2k_0)^{i\bar{\eta}^2} = \beta \tilde{E}_M(x_c, y_c)(x_0 - x_H)^{-i\bar{\eta}^2}$$

•Match to the outgoing H field, in the (k_x, y) representation.

$$E_H(k_x, y) = \tilde{E}_H(-k_0, y)[\exp i\theta_H(k_x, y)]\left(\frac{k_x + k_0}{k_x - k_0}\right)^{-i\bar{\eta}^2}$$

(This solution is exact between the two conversion regions.)

Matching at the second conversion

Follow a similar procedure in the vicinity of the second conversion, where the H rays intersect the M dispersion surface.

The outgoing fields from the second conversion are the 'reflected' M wave and the transmitted H wave; the latter propagates to high k_x and is absorbed via Landau damping and/or gyroresonance.

The fate of the transmitted M wave

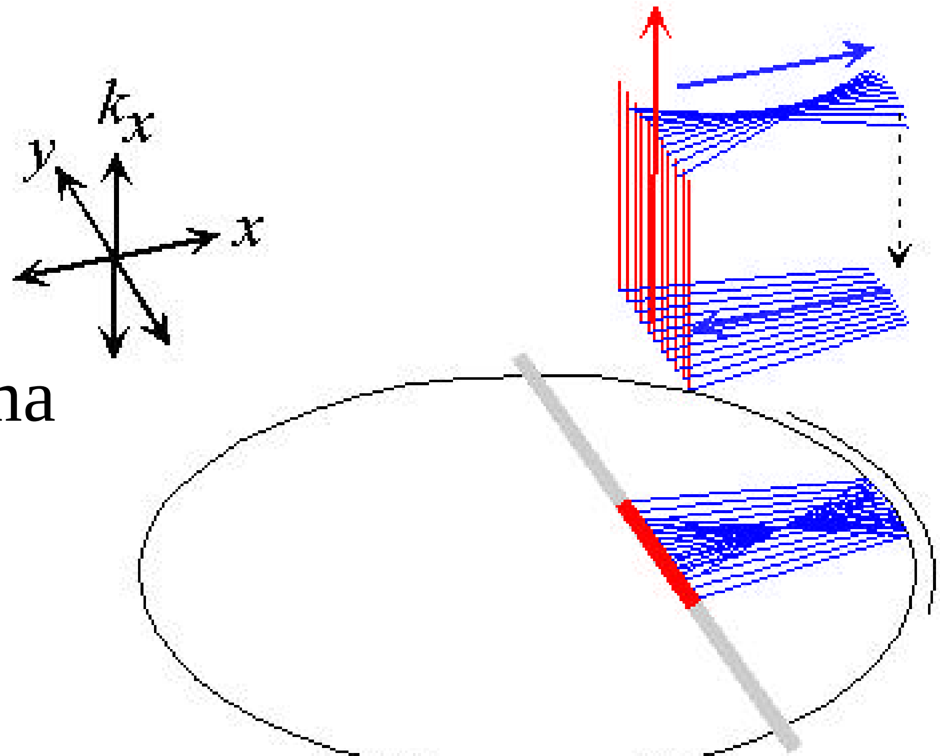
After transmission through the first conversion, the manifold of M rays is reflected from the plasma boundary at the inside edge of the tokamak. It then re-enters the hybrid resonance region, partially converts to a manifold of H rays, and is partially transmitted as a manifold of ‘reflected’M rays. Note, however, that these ray manifolds are different from those we have illustrated previously. (Although the previous H ray manifold and this new H ray manifold both lie in the same H dispersion surface, they have different phase functions and propagate independently). These are the first of two infinite sets of ray manifolds, both H and M.

Caveat emptor:

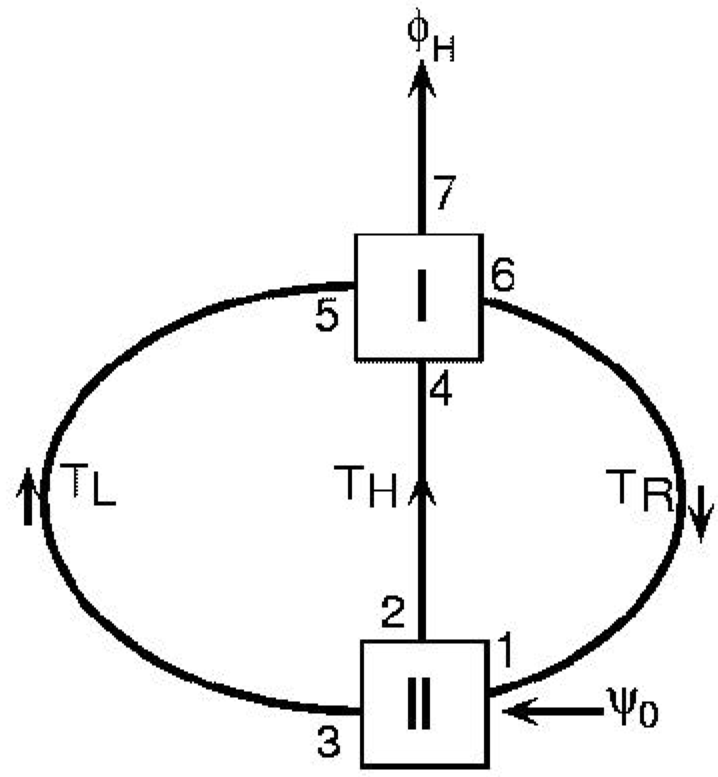
This approach is essentially that of geometric optics (but in a much more complicated setting than is customary). It omits diffraction effects, whose importance is yet to be determined.

Iterative conversion

The two 'reflected' M ray manifolds are, in turn, reflected inward at the outer plasma boundary. They then cross the resonance layer again, *déjà vu*...



In this diagram of the logic of iterated mode conversion, each ‘ray’ represents an infinite set of ray manifolds. The symbols (T_R, T_L, T_H) represent the free propagation of the waves from one conversion region to the other.



Numerical results

- Launch 1,000 rays focused on magnetic axis
- η =1, therefore weak transmission

$$\tau = e^{-\pi\eta^2} \approx 4\%$$

- follow rays through 100 resonance crossings
- superpose disturbances at output of upper conversion

Summary and conclusions

We have formulated an algorithm for calculating the wave field from iterated ray manifolds. The next step is implementation, and then comparison with full wave simulations.