Mode conversion in tokamak geometry

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Goals

- Compute global response of a tokamak to magnetosonic driving, including ion-hybrid conversion.
- Find spatial absorption profile and cavity Q.

Basic physical model

- Cold D-T plasma ($n_D=n_T=n_e/2$)
- $\mathbf{E}(x,z;t) = e^{-i\omega t}\mathbf{E}(x,z)$
- $\mathbf{B}(x) = B_0(1-x/L_B)\hat{y}$
- $\underline{\underline{D}}(x,z;-i\partial_x,-i\partial_z;\omega)\mathbf{E}(x,z)=0$ 2X2 cold plasma disp. tensor.
- Project onto 2 *uncoupled* polarizations:

$$\hat{e}_M \equiv \frac{\hat{x}+i\hat{z}}{\sqrt{2}}; \qquad \hat{e}_H \equiv \hat{x}$$

$$\mathbf{E}(x,z) = E_M \hat{e}_M + E_H \hat{e}_H \equiv \psi(x,z)\hat{e}_M + \phi(x,z)\hat{e}_H$$

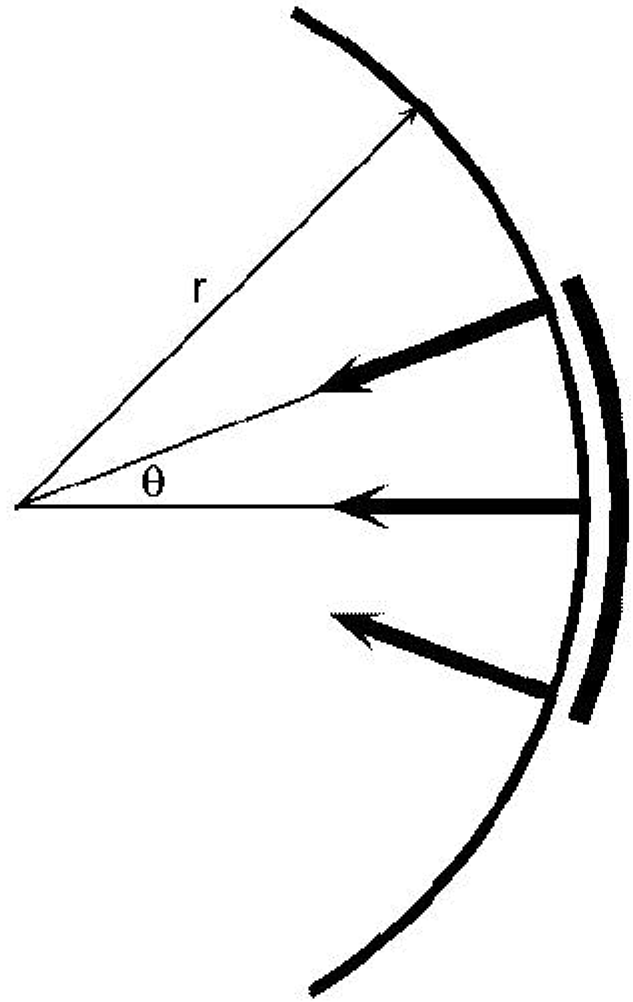
$$\begin{pmatrix} \hat{D}_H & \eta \\ \eta^* & \hat{D}_M \end{pmatrix} \begin{pmatrix} \phi \\ \psi \end{pmatrix} = 0$$

$$D_M = R - \tfrac{1}{2} N^2 = \frac{\omega^2}{c_A^2} - k_\perp^2 ; \quad D_H = x - x_H(\omega)$$

$$\eta = R/\sqrt{2}$$ (omitting constant factors)

The tokamak as a resonant cavity

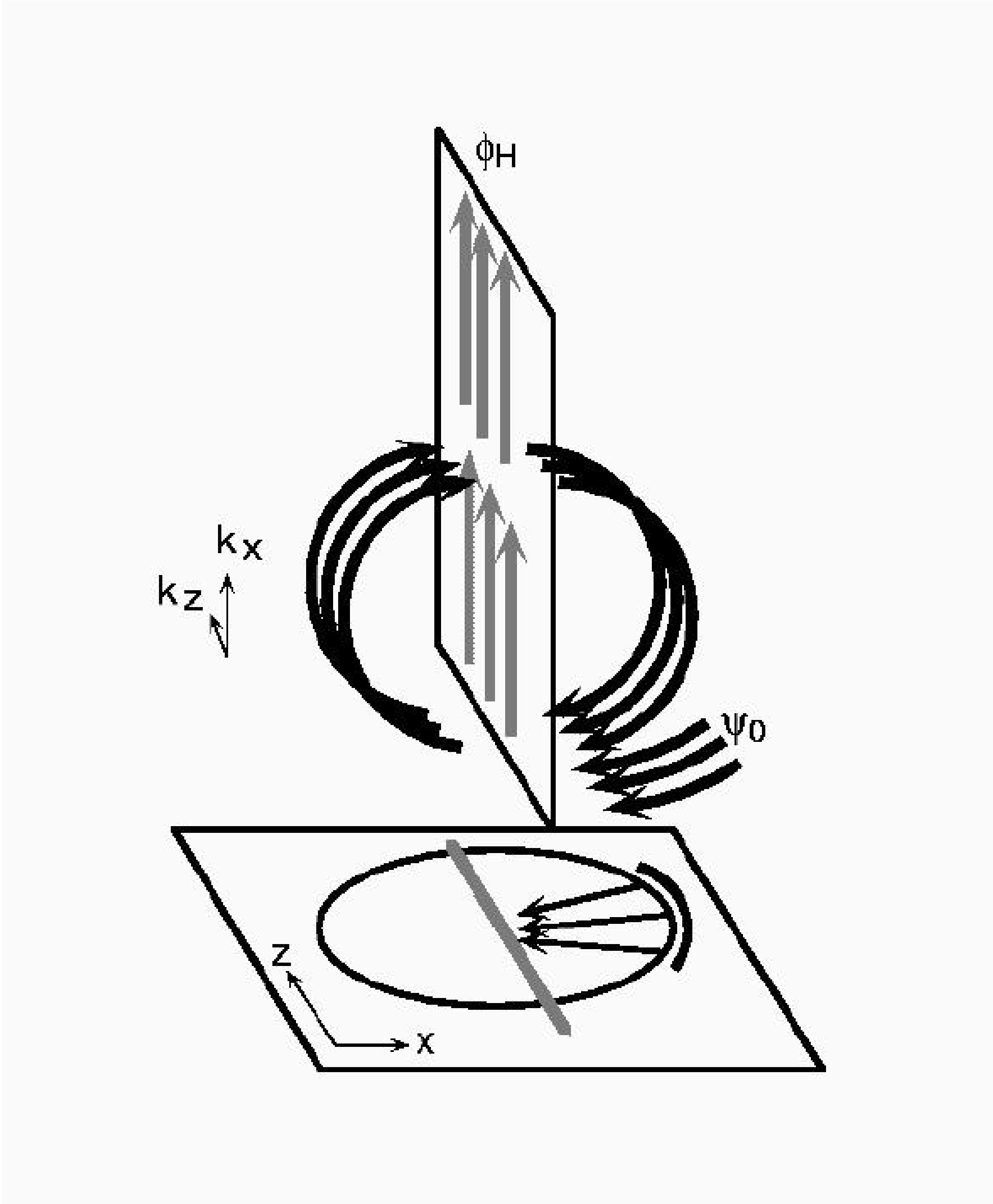
- Launch a family of rays from the antenna. Magnetosonic (MS) wave field injected is ψ_0.
- Calculate amplitude and phase transport using standard phase integral and van Vleck determinant
- At each resonance crossing use local modular approach to break the crossing into a two-step conversion process


r
θ

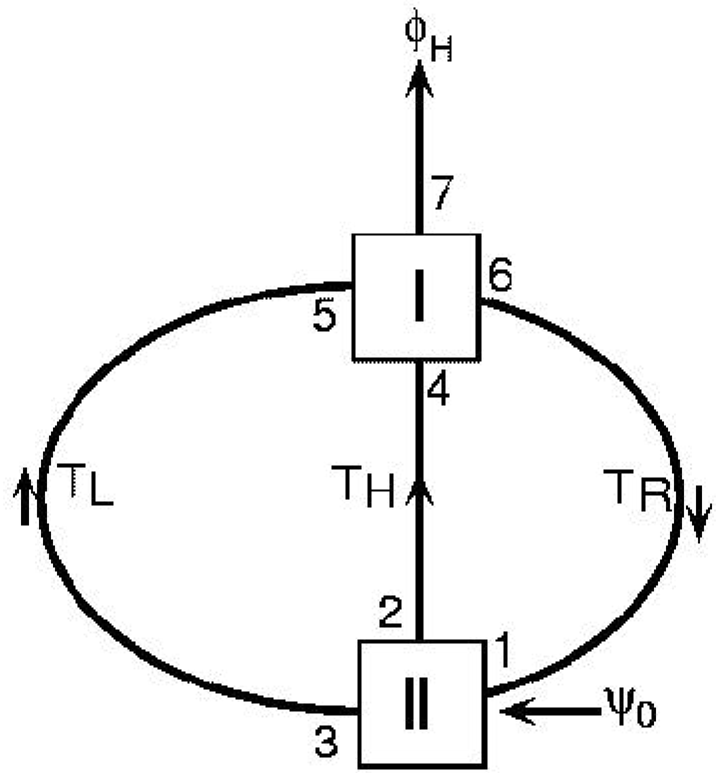
- At each conversion use S-matrix connection formula:

$$\begin{pmatrix} \phi^{(out)} \\ \psi^{(out)} \end{pmatrix} = \begin{pmatrix} \tau & \beta \\ -\beta^* & \tau \end{pmatrix} \begin{pmatrix} \phi^{(in)} \\ \psi^{(in)} \end{pmatrix}$$

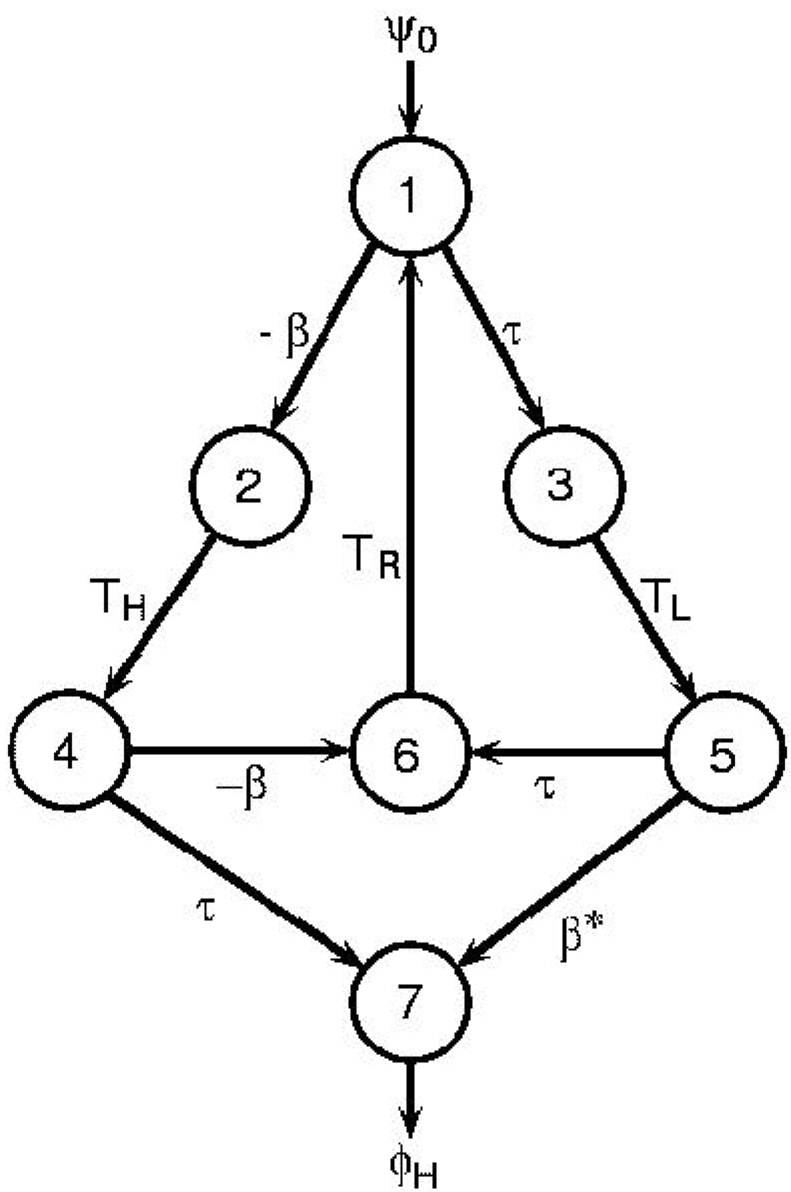
- At edge of plasma perform specular reflection of MS wave.
- Field escapes cavity via hybrid, ϕ, which eventually Landau damps on background.

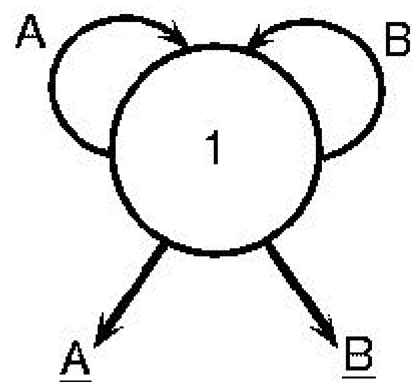


ϕ_H
k_x
k_z
ψ_0
z
x



ϕ_H
7
I
6
5
4
T_L
T_H
T_R
2
1
II
ψ_0
3


ψ0
1
- β
τ
2
3
TR
TH
TL
4
6
5
–β
τ
τ
β*
7
φH


A
B
1
A
B

Computing the cavity response: sum over all paths

- ‘Simple escape’: $\underline{A}$ $\underline{B}$
- ‘One loop’: $\underline{A}A$ $\underline{A}B$ $\underline{B}A$ $\underline{B}B$
- ‘L loops’: $(\underline{A}+\underline{B})(A+B)^L$
- Superpose all possible paths

$$\phi = (\underline{A}+\underline{B})\sum_{L=0}^{\infty}(A+B)^L\psi_0 \equiv \hat{C}(\omega)\psi_0$$

Numerical results

- Launch 1,000 rays focused on magnetic axis
- η =1, therefore weak transmission

$$\tau = e^{-\pi\eta^2} \approx 4\%$$

- follow rays through 100 resonance crossings
- superpose disturbances at output of upper conversion

Summary and conclusions

- Modular treatment of ion-hybrid resonance has been extended to 2D poloidal plane of the tokamak
- semi-classical treatment of propagation gives insight into energy flow through system
- 1000 rays through 100 bounces takes about 1 min. on a desktop workstation (SGI O2)